# Urban development dynamics and zoning

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Capital Theory
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# What are the transition dynamics associated with shocks in a city?

Quantitative urban models increasingly used to analyze shocks in cities

Ahlfeldt et al. (2015), Tsivanidis (2023), Dingel & Tintelnot (2024)

These are long-run models. Transition may be slow due to sluggish housing adjustment.

We develop a quantitative, dynamic, spatial model of housing development

- Housing is durable: easier to 'build up' than 'build down'

  Glaeser & Gyourko (2005)

- Frictions like zoning restrictions impede development

  Saiz (2010), Baum Snow & Han (2024)

Application: the dynamics of housing development in San Francisco.

- Methodology to uncover latent zoning restrictions from observed allocations
- Study transition dynamics associated with demand shocks and 'upzoning.'

# Model Outline: Households consume and reside, devs build housing

Time $t \in [0, \infty)$ and space $x \in [0, 1]$ are continuous.

At each $t$, mass of households decide where to live $x$, work at final goods firm, earn $w$.

- Choose floorspace and goods demands $h, c$, face relative floorspace price $p(x, t)$
- Freely mobile $\implies$ spatial eq'm holds at each instant

Developers make dynamic decisions

- Developers take housing prices as given
- Construct and operate floorspace $H$ subject to operating and adjustment costs

Landlords rent land to developers, consume final good.

Zoning regulations limit the amount of floorspace per unit land.

# Households consume goods and housing, and prefer amenities

Mass $N$ households.

At each $t$, households at $x$ maximize,

$$u(x,t) \equiv \max_{c,h} A(x,t) \left(\frac{c}{1-\beta}\right)^{1-\beta} \left(\frac{h}{\beta}\right)^{\beta} \quad \text{s.t.} \quad c + p(x,t)h \leq w(x,t)$$

Households are freely mobile, and choose $x$ to maximize $u(x,t)$.

A spatial equilibrium holds at all $t$,

$$\overline{u}(t) \equiv u(x,t) = u(x',t) \quad \forall x, x'$$

Allows for,

- Congestion externalities and spatial spillovers in $A(x,t)$.
- Commuting and labor market effects on $w(x,t)$. (Will do in quant. section)

# ‘Housing’ technology: costly to build or demolish fast

At each $x$, there is a developer that chooses how to develop floorspace. Developers compete for the right to operate on land, so land rents $r(x,t)$ exhaust all profit.

For a developer at $x$, the recursive form of their problem is given by the Hamilton-Jacobi-Bellman equation,

$$\rho\mathcal{V}(H,t) - \mathcal{V}_t(H,t) = \max_{\dot{H}} \left\{ p(x,t)H - \kappa(H) - \Phi(\dot{H}) + \mathcal{V}_H \dot{H} \right\}$$

- $\kappa(H)$: *operating costs*; $\kappa' > 0$, $\kappa'' > 0$
- $\Phi(\dot{H})$: *adjustment costs*, $\Phi$ increasing in $|\dot{H}|$, $\Phi'' > 0$ .

Can allow $\kappa$ and $\Phi$ to vary across space.

Durability: $\Phi(-\dot{H}) > \Phi(\dot{H})$ for $\dot{H} > 0$.

# Equilibrium

Given $A(x,t)$ and $w(x,t)$, an eq'm is paths $\{p(x,t), h(x,t), n(x,t), H(x,t)\}$ such that,

(i) given $w(x,t)$ and $p(x,t)$ households maximize utility: floorspace $h(x,t)$ is optimally demanded, and there is no opportunity for spatial arbitrage, $u(x,t) = u(x',t) = \bar{u}(t)$;

(ii) Total population is constant: $\int_0^1 n(x,t)dx = N$;

(iii) given $p(x,t)$, developers solve their HJB, optimally choosing floorspace paths $H(x,t)$;

(iv) rental prices $r(x,t)$ exhaust all development profit;

(v) housing markets clear at all locations and times, $H(x,t) = n(x,t)h(x,t)$.

Spatial equilibrium pins down prices, $p(x,t) = \left(\bar{u}(t)^{-1}A(x,t)w(x,t)\right)^{1/\beta}$.

## Lemma

*A unique static equilibrium ($\Phi = 0$) exists if $\kappa' > 0$ is inevitable and $\kappa'' \geq 0$.*

## Lemma

*The steady-state of the dynamic equilibrium is the static equilibrium.*

# Equilibrium characterization

Envelope condition + time-differentiated optimality condition, the solution to the developer's problem is a second-order ODE in $H$,

$$\rho\Phi'(\dot{H}) - \Phi''(\dot{H})\ddot{H} = p(x,t) - \kappa'(H)$$

If by a fixed $T$, $p(x,t) \to p(x,T)$, then this is a boundary value problem (BVP)

$$H(0) = H_0, \quad p(x,T) = \kappa'(H(T))$$

## Theorem

*For an arbitrary price path $p(x,t)$, the developer's BVP has a unique solution.*

## Proof.

Following Keller (1966), we show boundary value problems of the form $\ddot{y} = f(y, \dot{y}, t)$, $y(0) = y_0$ and $y(T) = y_T$ have a unique solution $y \in \mathcal{C}^2$ provided $\partial f / \partial y > 0$ and $\left|\partial f / \partial \dot{y}\right| \leq M$ for some $M \geq 0$. These conditions are met if $\kappa'' > 0$ and $\left|\frac{\Phi'\Phi'''}{(\Phi'')^2}\right| \leq \tilde{M}$ for $\tilde{M} \geq 0$ (true for power functions). □

# Equilibrium as a PIDE

For expositional clarity, we now assume,

$$\kappa(H) = \frac{H^{1+1/\gamma}}{1+1/\gamma}, \quad \Phi(\dot{H}) = \frac{\xi}{2}\left(\dot{H}\right)^2$$

Then the equilibrium can be represented as a partial integro-differential equation,

$$\begin{aligned} \rho\xi\dot{H}(x,t) - \xi\ddot{H}(x,t) + H(x,t)^{1/\gamma} &= \left(w(x,t)A(x,t)\right)^{1/\beta}\bar{u}(t)^{-1/\beta}, \\ \bar{u}(t) &= \left(\frac{1}{\beta}\int_0^1 H(x,t)A(x,t)^{1/\beta}w(x,t)^{1/\beta-1}dx\right)^{\beta} \\ H(x,0) &= H_0(x) \\ \lim_{T\to\infty} H(x,T) &= H_\infty(x) \end{aligned}$$

where $H_\infty(x)$ is the steady-state solution.

# Model behavior: localized demand shock

- 3 identical locations
- unanticipated, permanent shock to location specific demand

$$A(1,0) \uparrow\uparrow$$
$$A(2,0) \uparrow,$$
$$A(3,0) \text{ fixed}$$

Demand shock at 1 ‘spills over’ to location 2 (e.g., access to amenities).

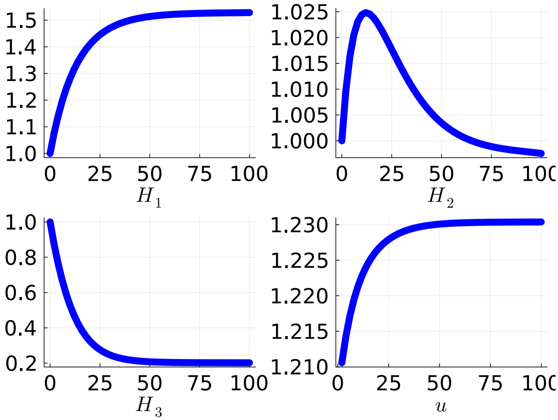


Figure 1: X-axis: time. Y-axis: % change in $H$.

# Model behavior: anticipation

- Announce shock some periods prior

Green: arrival of shock
Blue: original paths
Red: new paths

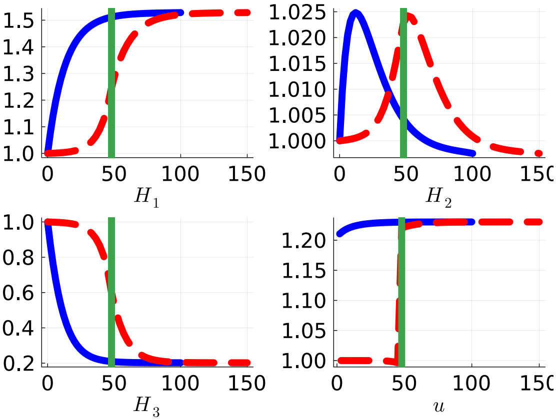


Figure 2: X-axis: time. Y-axis: % change in $H$.

# Model behavior: localized demand shock – durable housing

Now, suppose,

$$\Phi(\dot{H}) = \begin{cases} \frac{\xi_U}{2}\left(\dot{H}\right)^2 & \dot{H} \geq 0 \\ \frac{\xi_D}{2}\left(\dot{H}\right)^2 & \dot{H} < 0 \end{cases}$$

where $\xi_U << \xi_D$.

Blue: original paths
Red: new paths

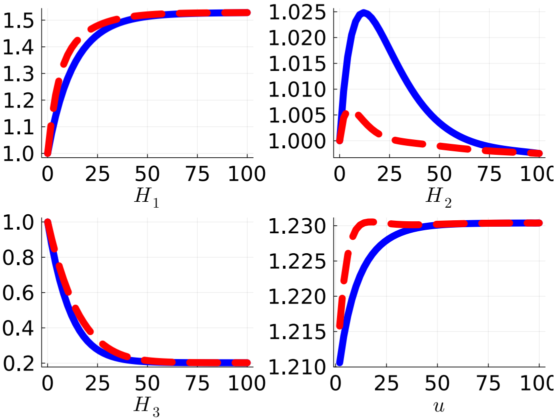


Figure 3: X-axis: time. Y-axis: % change in $H$.

# Model behavior: localized demand shock – zoning

Now, suppose, $H(1) \leq Z$.

Set $Z$ = 90% steady state $H(1)$ without zoning.

Blue: original paths
Red: new paths

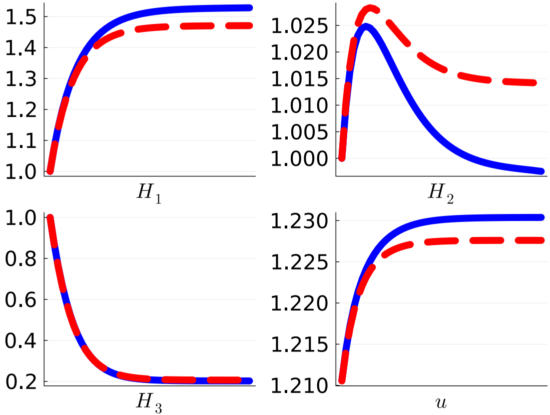


Figure 4: X-axis: time. Y-axis: % change in $H$.

# Quantification: San Francisco

**Goal**: Understand the transition dynamics associated with local demand and supply shocks:

1. Demand shock: ‘Return to downtown’ (wages $\uparrow$ in downtown)
2. Upzone western parts of the city in light rail corridor (actual policy proposal)

Data:

- Structure characteristics and zoning status (OpenSF + Municipal Code)
- Floorspace prices Pennington (2021)
- Residency and commuting patterns (LEHD-LODES)

# Quantitative model: additions

Two features:

1. Commuting
2. ‘Quantitatively tractable’ zoning

Commuting:

- Workplaces will differ in TFP
- Households will commute based on idiosyncratic shocks and iceberg commuting costs
- Allows us to recover wages with a gravity regression on commuting flows.

Commuting details

**Zoning**: Microfoundations + recovering restrictions from data

# Zoning is unobserved

In the model, zoning is a restriction, $H(x) \leq Z(x)$.

Problem: *de facto* zoning is unobserved

Parcels zoned RH-1...

> *"...are occupied almost entirely by single-family houses on lots 25 feet in width, without side yards. Floor sizes and building styles vary, but tend to be uniform within tracts developed in distinct time periods. Though built on separate lots, the structures have the appearance of small-scale row housing, rarely exceeding 35 feet in height. Front setbacks are common, and ground level open space is generous. In most cases the single-family character of these Districts has been maintained for a considerable time." §209.1, San Francisco Municipal Code*

# Zoning is unobserved – a model to recover zoning

**Goal**: A model of zoning restrictions that allows us to *infer* $Z(x)$ from realized allocations.

Locations $x$ have a measure of parcels $T(x)$. Each parcel $\omega$: can build floorspace with final good and land,

$$h_i(x,\omega) = e(x,\omega)c(x,\omega)^{\frac{\gamma}{1+\gamma}} T(x,\omega)^{\frac{1}{1+\gamma}}$$

$e(x,\omega) \sim G_i$, $G_i$ Pareto$(E(x),\varphi)$. Zoning restriction applies on a per-parcel basis,

$h(x,\omega) \leq Z(x)$. Developers take floorspace prices $p(x)$ and land prices $r(x)$ as given.

Solution to developer's problem: Develop most efficient parcels first,

$$\underbrace{H(x)}_{\text{total floorspace}} = T(x)\left[Z_i\big(1 - G_x(\tilde{e})\big) + \int_{E(x)}^{\tilde{e}} (p(x)e)^{\gamma} dG_x(e)\right], \quad \tilde{e}: \text{ cutoff productivity.}$$

**Implication**: Bunching in the building height distribution at $Z(x)$.

# Zoning data

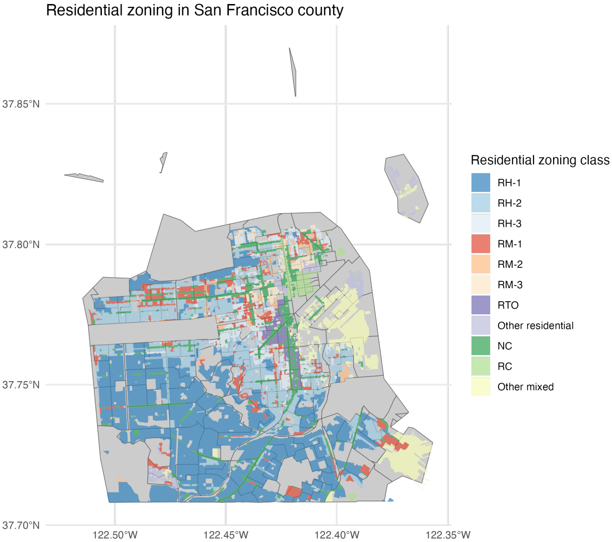


Figure 5: Residential Zoning classifications for SF

# Zoning data

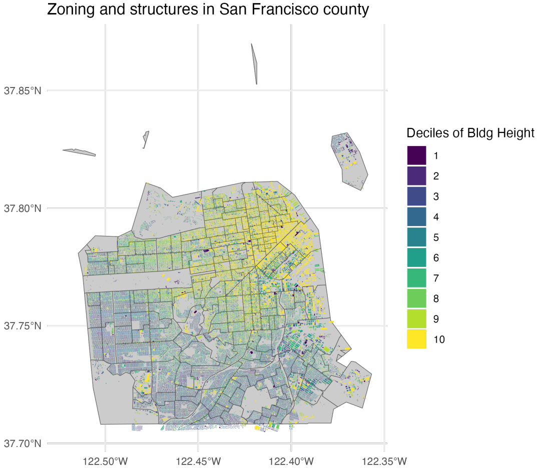


Figure 5: Building heights classifications for SF

# Bunching in the building height distribution - 'RKD'

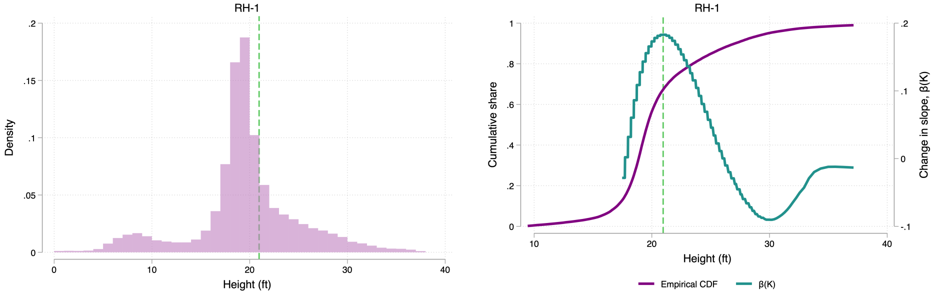


Figure 6: Left: Building Height Distribution. Right: Building Heights CDF for RH-1 and estimates of the kink in the derivative.

Estimate for parcel $i$ in zoning classification $C$ and cutoff $K$

$$\text{Percentile}_{i,C} = \alpha + \beta_C(K) \times (h_{i,C} - K) \times 1(h_{i,C} < K) + f(h_{i,c}, K) + u_{i,C}$$

Estimate, $\hat{Z}_C = \text{argmax}_K \hat{\beta}_C(K)$.

# Inversion

Data: locations are census tracts *i*. Given,

1. $S_i$ - share of parcels for which zoning binds,
2. $\mathbb{E}[w_i]$ (expected wages),
3. $N_i$ (population),
4. $p_i$ (floorspace prices)
5. Parameters, $\beta, \gamma, \epsilon, \varphi$

| $\beta$ | $\gamma$ | $\epsilon$ | $\varphi$ |
|---|---|---|---|
| 0.24 | 1.85 | 2.18 | 6 |

Can find unique $A_i$, $E_i$, $T_i$, and $Z_i$ that rationalize the data as a steady-state eq'm of the model.

$S_i$ $Z_i$ $A_i$ $\mathbb{E}[w_i]$

# Counterfactual: return to downtown

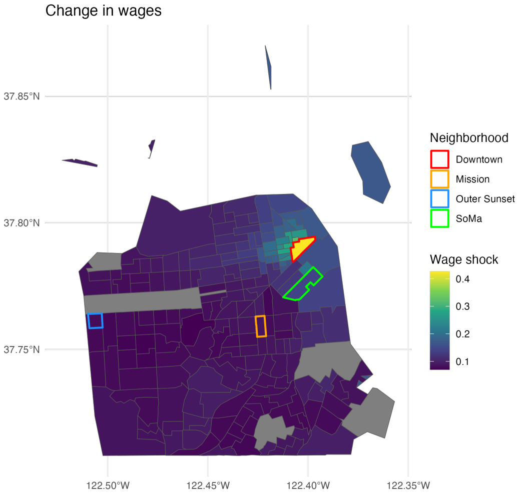


Figure 7: Geography of the wage shock

# Counterfactual: return to downtown

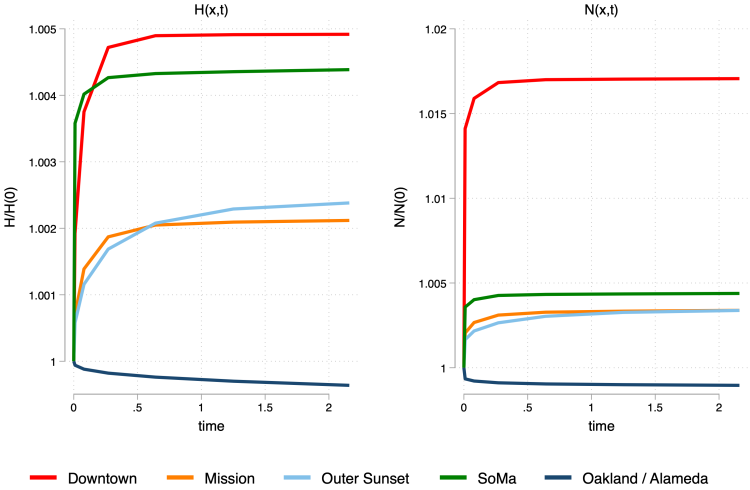


Figure 8: Wage shock – time series

# Counterfactual: upzone Sunset District Light Rail Corridor

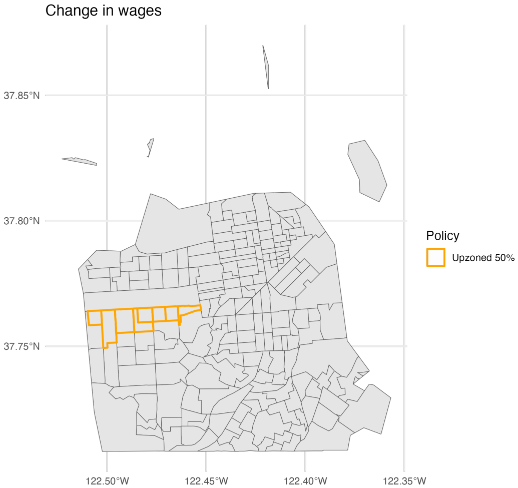


Figure 9: Locations upzoned

# Counterfactual: upzone Sunset District Light Rail Corridor

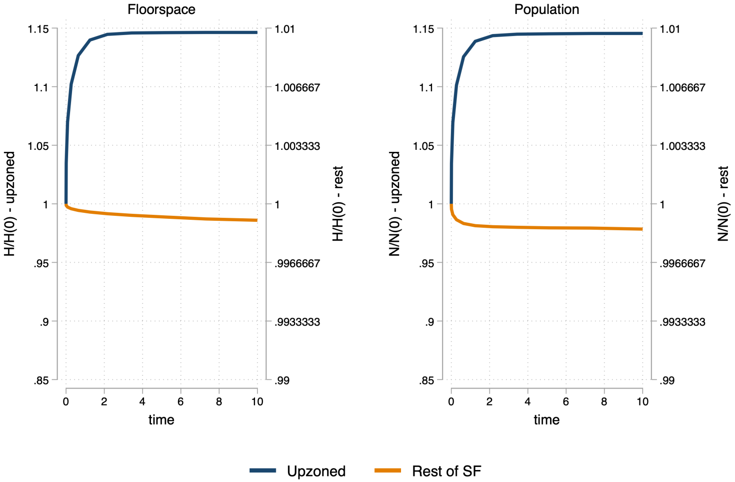


Figure 10: Locations upzoned

# Commuting – recover wages through gravity

- Households choose where to live $x$, based on expected wages $\mathbb{E}[w(x)]$
- After location choice, productivity shock in efficiency units of labor, $\nu(x') \sim$Frechet$(1, \epsilon)$ is realized
- Pay commuting iceberg costs as fraction of the wage, $\tau(x, x')$
- Workplace choice is then,

$$\max_{x'} w(x')\nu(x')/\tau(x, x') \implies \mathbb{E}[w(x') \mid x] \propto \left(\int_{x'} \left(w(x')/\tau(x, x')\right)^{\epsilon}\right)^{1/\epsilon}$$

- Firms all produce same freely traded final good with linear technology,

$$y(x') = Q(x')L(x') \implies w(x') = Q(x')$$

- Under this structure, can recover $\tau(x, x')$ and $Q(x')$ from a gravity regression on commuting flows.

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# Inverted $S_i$


Figure 11: Share bound by zoning

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# Inverted $Z_i$

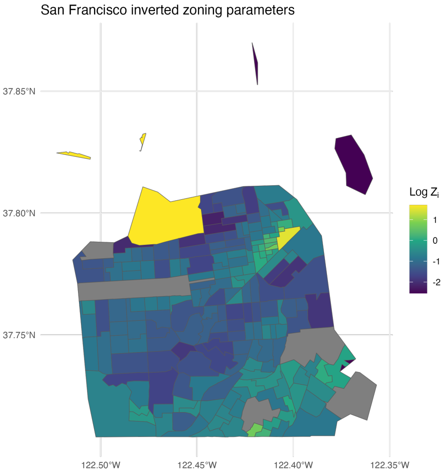


Figure 12: Recovered $Z_i$

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# Inverted $A_i$

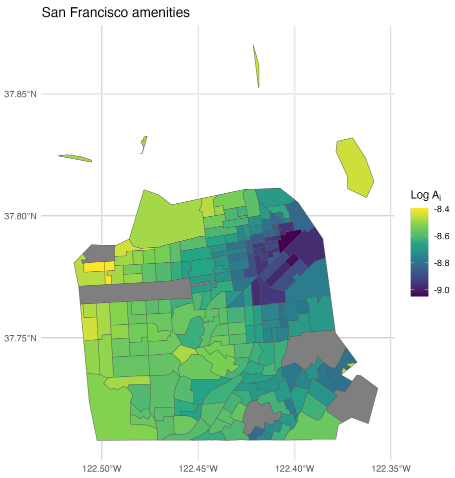


Figure 13: Recovered $A_i$

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# Inverted $\mathbb{E}[w_i]$

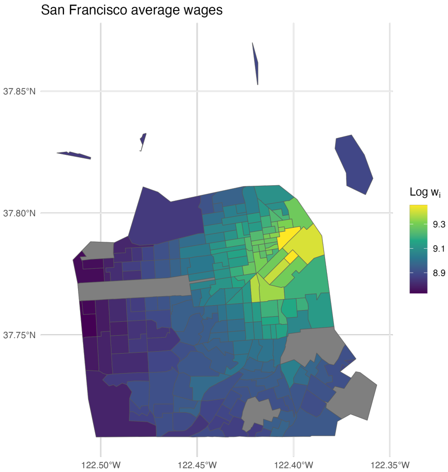


Figure 14: Recovered $\mathbb{E}[w_i]$

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