Urban development dynamics and zoning

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Urban Economics Association Conference
Berlin, 29 March 2025

What are the transition dynamics associated with shocks in a city?

Quantitative urban models increasingly used to analyze shocks in cities

Ahlfeldt et al. (2015), Tsivanidis (2023), Dingel & Tintelnot (2024)

These are long-run models. Transition may be slow due to sluggish housing adjustment.

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We develop a quantitative, dynamic, spatial model of housing development

- Housing is durable: easier to 'build up' than 'build down' Glaeser & Gyourko (2005)
- Frictions like zoning impede development Saiz (2010), Baum Snow & Han (2024)

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Application: the dynamics of housing development in San Francisco.

- Infer latent zoning restrictions from observed allocations Brueckner et al. (2024)
- Study transition dynamics associated with demand shocks and 'upzoning'

Main contribution and idea

Several frontiers for computing transition dynamics in spatial economies

- Caliendo / Dvorkin / Parro (2019)
  dynamic labor reallocation; ‘exact hat’ transition dynamics; no capital accumulation
- Bilal & Rossi-Hansberg (2024)
  1st order approximation to transition dynamics using the Master equation
- Parkhomenko et al. (2025)
  HANK + space with ‘mixed time’

What we do:

- Continuous time, $\approx$ continuous space, local capital accumulation (housing)
- Globally solve for the eq’m transition dynamics following shocks

Key idea: state space huge ($> N \times T$)...
...but spatial equilibrium holds at any instant $\rightarrow$ massive reduction in dimensionality!

Model Outline: Households consume and reside, devs build housing

Time $t \in [0, \infty)$ and space $x \in [0, 1]$ are continuous.

At each t, mass of households decide where to live x and work; earn $w(x, t)$.

- Choose floorspace and goods demands h, c, face relative floorspace price $p(x, t)$
- Freely mobile $\implies$ spatial eq'm holds at each instant

Developers make dynamic decisions

- Developers take housing prices as given
- Construct and operate floorspace H subject to operating and adjustment costs

Landlords rent land to developers, consume final good.

Zoning regulations limit the amount of floorspace per unit land.

Households preferences: super standard

Mass N households.

At each t, households at x maximize,

$$u(x,t) \equiv \max_{c,h} A(x,t) \left(\frac{c}{1-\beta}\right)^{1-\beta} \left(\frac{h}{\beta}\right)^{\beta} \quad \text{s.t.} \quad c + p(x,t)h \leq w(x,t)$$

Households are freely mobile, and choose x to maximize $u(x,t)$. Population: $n(x,t)$.

A spatial equilibrium holds at all t, $\overline{u}(t) \equiv u(x,t) = u(x',t) \quad \forall x, x'$.

With Fréchet shocks, analagous object is ex-ante expected utility. See paper.

Allows for,

- Congestion externalities in $A(x,t)$.
- Commuting and labor market effects on $w(x,t)$.

‘Housing’ technology: costly to build or demolish fast

Developers at each x, choose how to develop floorspace.

Developer’s Hamilton-Jacobi-Bellman equation,

$$\rho\mathcal{V}(x,H,t) - \mathcal{V}_t(x,H,t) = \max_{\dot{H}} \left\{ p(x,t)H - \kappa(H) - \Phi(\dot{H}) + \mathcal{V}_H \dot{H} \right\}$$

Sequence space formulation, given $p(x,t)$ path.

- $\kappa(H)$: *operating costs*; $\kappa' > 0$, $\kappa'' > 0$
- $\Phi(\dot{H})$: *adjustment costs*, Φ increasing in $|\dot{H}|$, $\Phi'' > 0$.

Can allow κ and Φ to vary across space.

Durability: $\Phi(-\dot{H}) > \Phi(\dot{H})$ for $\dot{H} > 0$.

‘Housing’ technology: costly to build or demolish fast

Developers at each x, choose how to develop floorspace.

State-space formulation (the master equation),

$$\rho\mathcal{V}(x,H,\mathcal{S}) = \max_{\dot{H}} \left\{ \mathcal{P}(x,\mathcal{S})H - \kappa(H) - \Phi(\dot{H}) + \mathcal{H}(x,\mathcal{S})[\mathcal{V}] + \int \frac{\partial\mathcal{V}}{\partial S}(x,x')\mathcal{H}^*(x',\mathcal{S})[\mathcal{S}]dx' \right\}$$

$\mathcal{H}$: continuation value operator, $\mathcal{H}^*$: its adjoint (KFE).

State $\mathcal{S} = (H, A, n)$ is the spatial distribution of H, A, n.

$\mathcal{S}$ sufficient to forecast prices through $\mathcal{P}(x,\mathcal{S})$.

This is hard: developers need to know the full evolution of $\mathcal{S}$ to forecast prices!

Equilibrium

Given $A(x,t)$ and $w(x,t)$, an eq'm is paths $\{p(x,t), h(x,t), n(x,t), H(x,t)\}$ such that,

(i) given $w(x,t)$ and $p(x,t)$ households optimally choose floorspace $h(x,t)$ and $c(x,t)$,

(ii) there is no opportunity for spatial arbitrage, $u(x,t) = u(x',t) = \bar{u}(t)$;

(iii) Total population is constant: $\int_0^1 n(x,t)dx = 1$;

(iv) given $p(x,t)$, developers solve their HJB, optimally choosing floorspace paths $H(x,t)$;

(v) housing markets clear at all locations and times, $H(x,t) = n(x,t)h(x,t)$.

Key: Spatial equilibrium pins down prices, $p(x,t) = \left(\bar{u}(t)^{-1}A(x,t)w(x,t)\right)^{1/\beta}$.

Lemma

A unique static equilibrium ($\Phi = 0$) exists if $\kappa' > 0$ is invertible and $\kappa'' \geq 0$.

Lemma

The steady-state of the dynamic equilibrium is the static equilibrium.

Equilibrium characterization – how do we make any progress?

Developer's problem solution is a second-order ODE in H,

$$\rho\Phi'(\dot{H}) - \Phi''(\dot{H})\ddot{H} = p(x,t) - \kappa'(H)$$

(Get by envelope condition + time-differentiated optimality condition).

If by T we reach a steady state, then this is a boundary value problem (BVP):

$$H(x,0) = H_0(x), \quad p(x,T) = \kappa'(\underbrace{H_\infty(T)}_{\text{steady state solution}})$$

Theorem

For an arbitrary price path $p(x,t)$, the developer's BVP has a unique solution.

Proof.

Following Keller (1966), we show boundary value problems of the form $\ddot{y} = f(y,\dot{y},t)$, $y(0) = y_0$ and $y(T) = y_T$ have a unique solution $y \in \mathcal{C}^2$ provided $\partial f/\partial y > 0$ and $\left|\partial f/\partial \dot{y}\right| \leq M$ for some $M \geq 0$. These conditions are met if $\kappa'' > 0$ and $\left|\frac{\Phi'\Phi'''}{(\Phi'')^2}\right| \leq \tilde{M}$ for $\tilde{M} \geq 0$ (true for power functions). □

Equilibrium as a integro-differential system

For expositional clarity, we now assume,

$$\kappa(H) = \frac{H^{1+1/\gamma}}{1+1/\gamma}, \quad \Phi(\dot{H}) = \frac{\xi}{2}(\dot{H})^2$$

Then the equilibrium can be represented as an integro-differential system,

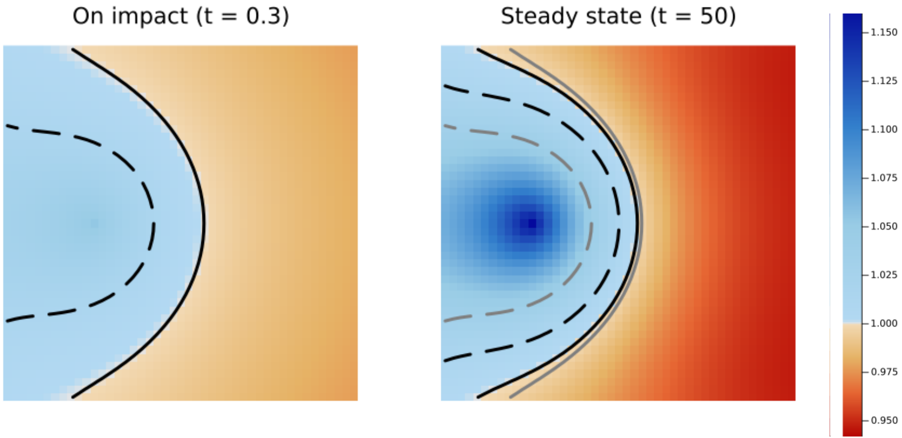
$$\begin{aligned}
\rho\xi\dot{H}(x,t) - \xi\ddot{H}(x,t) + H(x,t)^{1/\gamma} &= \big(w(x,t)A(x,t)\big)^{1/\beta}\bar{u}(t)^{-1/\beta}, \\
\bar{u}(t) &= \left(\frac{1}{\beta}\int_0^1 H(x,t)A(x,t)^{1/\beta}w(x,t)^{1/\beta-1}dx\right)^{\beta} \\
H(x,0) &= H_0(x) \\
\lim_{T\to\infty} H(x,T) &= H_\infty(x)
\end{aligned}$$

Key: $\bar{u}$ is a 'sufficient statistic' for $\mathcal{S}$.

Fréchet shocks | Exogenous commuting | Labor market | Neoclassical investment

Full dynamics: $H(t)/H(0)$ after a wage shock.

50% wage shock at (0.25,0.5), diffuses across space through commuting network



$N = 2,025$, $T = 50$, < 1min runtime, < 185 lines of julia code.

Solid line is 0% contour; dashed 2.5%

Model behavior: localized demand shock

- 3 identical locations
- unanticipated, permanent shock to location specific demand

$A(1,0) \uparrow\uparrow$

$A(2,0) \uparrow$,

$A(3,0)$ fixed

Demand shock at 1 ‘spills over’ to location 2 (e.g., access to amenities).

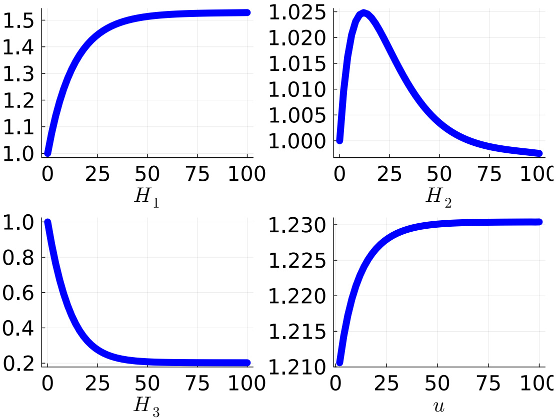


Figure 1: X-axis: time. Y-axis: % change in H.

Model behavior: anticipation

- Announce shock some periods prior

Green: arrival of shock
Blue: original paths
Red: new paths

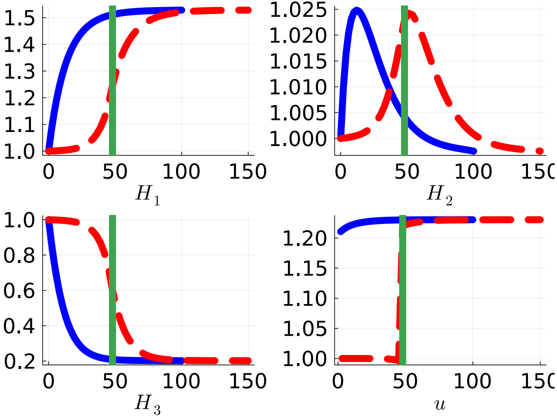


Figure 2: X-axis: time. Y-axis: % change in *H*.

Model behavior: localized demand shock – durable housing

Now, suppose,

$$\Phi(\dot{H}) = \begin{cases} \frac{\xi_U}{2}\left(\dot{H}\right)^2 & \dot{H} \geq 0 \\ \frac{\xi_D}{2}\left(\dot{H}\right)^2 & \dot{H} < 0 \end{cases}$$

where $\xi_U << \xi_D$.

Blue: original paths
Red: new paths

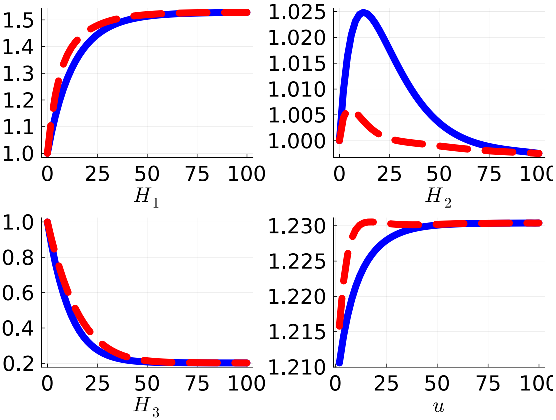


Figure 3: X-axis: time. Y-axis: % change in H.

Model behavior: localized demand shock – zoning

Now, suppose, $H(1) \leq Z$.

Set $Z = 90\%$ steady state $H(1)$ without zoning.

Blue: original paths
Red: new paths

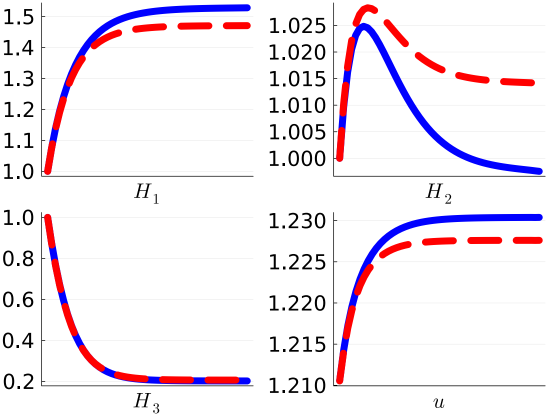


Figure 4: X-axis: time. Y-axis: % change in H.

Quantification: San Francisco

Goal: Understand the transition dynamics associated with local demand and supply shocks:

1. Demand shock: 'Return to downtown' (wages $\uparrow$ in downtown)
2. Upzone western parts of the city in light rail corridor (actual policy proposal)

Data:

- Structure characteristics and zoning status (OpenSF + Municipal Code)
- Floorspace prices Pennington (2021)
- Residency and commuting patterns (LEHD-LODES)

Quantitative model: additions

Two features:

1. Commuting
2. ‘Quantitatively tractable’ zoning

Commuting:

- Workplaces will differ in TFP
- Households will commute based on idiosyncratic shocks and iceberg commuting costs
- Allows us to recover wages with a gravity regression on commuting flows.

Commuting details

Zoning: Microfoundations + recovering restrictions from data

Zoning is unobserved

In the model, zoning is a restriction, $H(x) \leq Z(x)$.

Problem: *de facto* zoning is unobserved

Parcels zoned RH-1...

> *"...are occupied almost entirely by single-family houses on lots 25 feet in width, without side yards. Floor sizes and building styles vary, but tend to be uniform within tracts developed in distinct time periods. Though built on separate lots, the structures have the appearance of small-scale row housing, rarely exceeding 35 feet in height. Front setbacks are common, and ground level open space is generous. In most cases the single-family character of these Districts has been maintained for a considerable time." §209.1, San Francisco Municipal Code*

Zoning is unobserved – a model to recover zoning

Goal: A model of zoning restrictions that allows us to *infer* $Z(x)$ from realized allocations.

Locations x have a measure of parcels $T(x)$. Each parcel ω: can build floorspace with final good and land,

$$h_i(x,\omega) = e(x,\omega)c(x,\omega)^{\frac{\gamma}{1+\gamma}} T(x,\omega)^{\frac{1}{1+\gamma}}$$

$e(x,\omega) \sim G_i$, G_i Pareto$(E(x),\varphi)$. Zoning restriction applies on a per-parcel basis,

$h(x,\omega) \leq Z(x)$. Developers take floorspace prices $p(x)$ and land prices $r(x)$ as given.

Solution to developer's problem: Develop most efficient parcels first,

$$\underbrace{H(x)}_{\text{total floorspace}} = T(x)\left[Z_i\big(1 - G_x(\tilde{e})\big) + \int_{E(x)}^{\tilde{e}} (p(x)e)^{\gamma} dG_x(e)\right], \quad \tilde{e}: \text{ cutoff productivity.}$$

Implication: Bunching in the building height distribution at $Z(x)$.

Zoning data

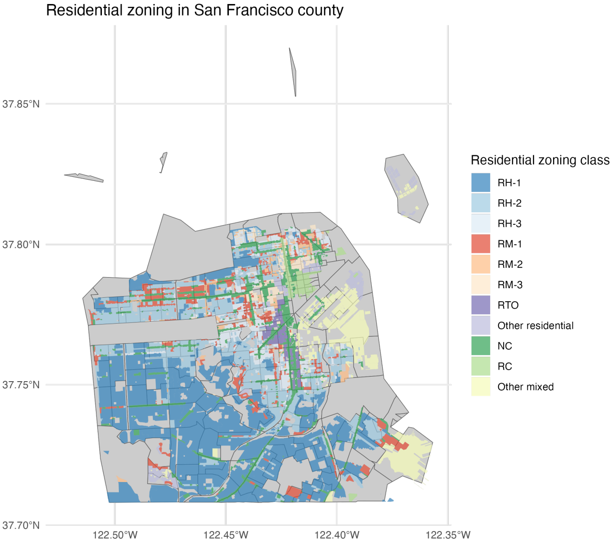


Figure 5: Residential Zoning classifications for SF

Zoning data

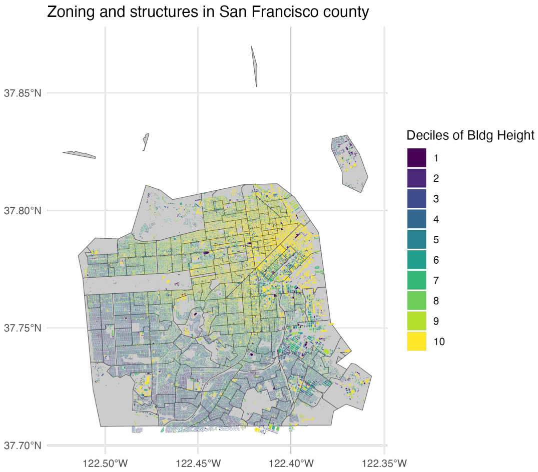
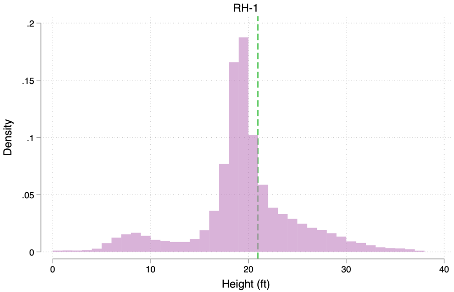


Figure 5: Building heights classifications for SF

Bunching in the building height distribution - ‘RKD’



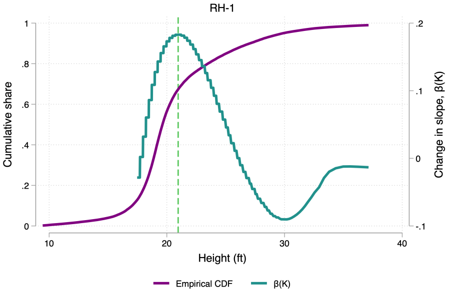


Figure 6: Left: Building Height Distribution. Right: Building Heights CDF for RH-1 and estimates of the kink in the derivative.

Estimate for parcel i in zoning classification C and cutoff K

$$\text{Percentile}_{i,C} = \alpha + \beta_C(K) \times (h_{i,C} - K) \times 1(h_{i,C} < K) + f(h_{i,c}, K) + u_{i,C}$$

Estimate, $\hat{Z}_C = \text{argmax}_K \hat{\beta}_C(K)$.

Inversion

Data: locations are census tracts i. Given,

1. S_i - share of parcels for which zoning binds,
2. $\mathbb{E}[w_i]$ (expected wages),
3. N_i (population),
4. p_i (floorspace prices)
5. Parameters, $\beta, \gamma, \epsilon, \varphi$

β	γ	ϵ	φ
0.24	1.85	2.18	6

Can find unique A_i, E_i, T_i, and Z_i that rationalize the data as a steady-state eq'm of the model.

S_i Z_i A_i $\mathbb{E}[w_i]$

Counterfactual: return to downtown

2.5% wage increase in SF's CBD.

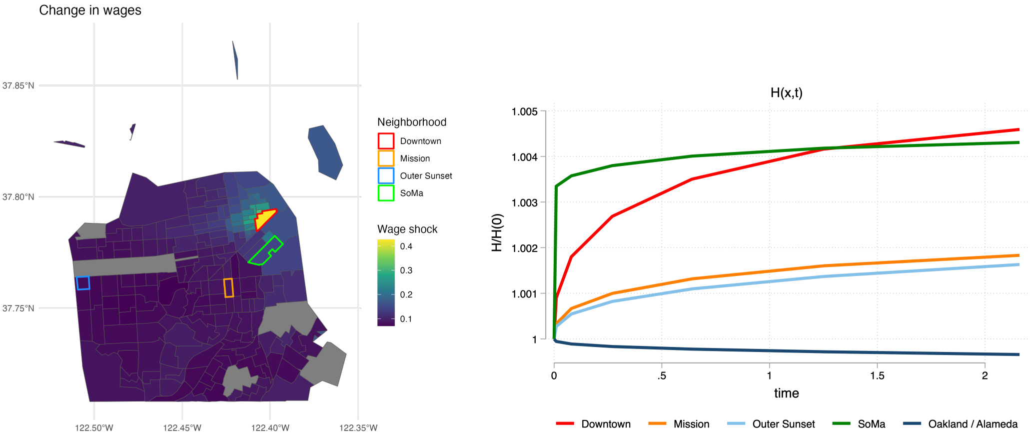
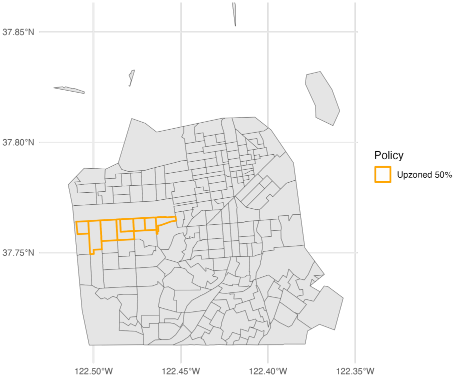


Figure 7: Left: geography of the wage shock in San Francisco. Right: time paths of the evolution of the housing stock in some select neighborhoods.

Counterfactual: upzone Sunset District Light Rail Corridor



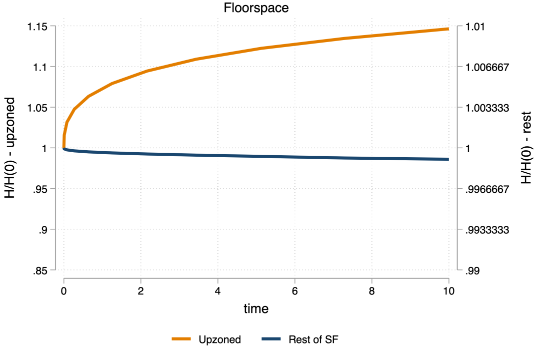


Figure 8: Left: the upzoned tracts. Right: time paths of the evolution of the housing stock in the upzoned neighborhoods and the rest of San Francisco.

Thank you!

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Handling Frechet shocks

Suppose location choice for each agent solves,

$$\max_x u(x,t)\vartheta(x,t)$$

where $\vartheta(x,t)$ is iid 'Frechet' w/ shape $1/\theta$. Define,

$$\mathcal{W}(t) = \left(\int_0^1 \left(u(x,t)\right)^{\theta} dx\right)^{1/\theta}$$

Then,

$$p(x,t) = \left(\frac{\beta w(x,t)}{H(x,t)}\right)^{\frac{1}{1+\theta\beta}} \left(\frac{A(x,t)w(x,t)}{\mathcal{W}(t)}\right)^{\frac{\theta}{1+\theta\beta}}$$

so,

$$H(x,t)^{\frac{1}{1+\theta\beta}} \left(-\xi \ddot{H}(x,t) + \rho\xi H(x,t) + H(x,t)^{1/\gamma}\right) = \tilde{\beta}\left(\frac{A(x,t)w(x,t)^{1+1/\theta}}{\mathcal{W}(t)}\right)^{\frac{\theta}{1+\theta\beta}}$$

Goes to original system as $\theta \to \infty$. Back to eq

Endogenous wages?

Differentiated goods, CES demand w/ EoS σ, *free trade*, tech: $y(x,t) = Q(x,t)L(x,t)$

Wages pinned down through market clearing,

$$w(x,t)n(x,t) = (1-\beta)\int_0^1 \left(w(x,t)Q(x,t)\right)^{1-\sigma} w(s,t)n(s,t)ds$$

Define city GDP,

$$\mathcal{Y}(t) = \frac{1}{\beta}\int_0^1 p(x,t)H(x,t)dx$$

Wages can be expressed,

$$w(x,t) = Q(x,t)\left(\tilde{\beta}\frac{A(x,t)^{1/\beta}H(x,t)}{\mathcal{Y}(t)\mathcal{U}(t)^{1/\beta}}\right)^{\frac{-1}{1/\beta+(\sigma-1)}}$$

$\rightarrow$ Now need $\mathcal{U}(t)$ and $\mathcal{Y}(t)$ to forecast prices. Back to original system as $\sigma \rightarrow \infty$ Back to eq

Neoclassical investment

Suppose developer problem is,

$$\rho\mathcal{V}(x,H,t) - V_t(x,H,t) = \max_I \left\{ p(x,t)H(x,t) - \underbrace{\tilde{\Phi}(I,H)}_{\text{adj costs}} + V_H \underbrace{(I - \delta H)}_{\text{LOM on } H} \right\}$$

δ= depreciation rate.

Convex cost of investment $\tilde{\Phi}(I,H)$

'operating costs' $\sim$ offsetting depreciation with investment.

Developer ODE,

$$-\xi\ddot{H}(x,t) + \rho\xi\dot{H}(xt) + \xi\delta(\rho+\delta)H(x,t) = p(x,t).$$

With quadratic investment costs. Housing demand side the same.

Back to eq

Commuting – recover wages through gravity

- Households choose where to live x, based on expected wages $\mathbb{E}[w(x)]$
- After location choice, productivity shock in efficiency units of labor, $\nu(x') \sim$Frechet$(1, \epsilon)$ is realized
- Pay commuting iceberg costs as fraction of the wage, $\tau(x, x')$
- Workplace choice is then,

$$\max_{x'} w(x')\nu(x')/\tau(x, x') \implies \mathbb{E}[w(x') \mid x] \propto \left(\int_{x'} \left(w(x')/\tau(x, x') \right)^{\epsilon} \right)^{1/\epsilon}$$

- Firms all produce same freely traded final good with linear technology,

$$y(x') = Q(x')L(x') \implies w(x') = Q(x')$$

- Under this structure, can recover $\tau(x, x')$ and $Q(x')$ from a gravity regression on commuting flows.

Back to quant Back to eq

Inverted S_i

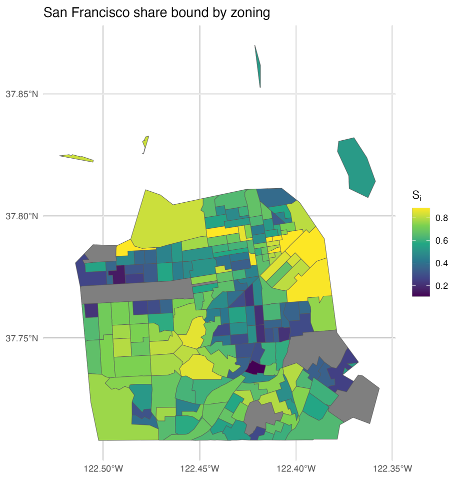

Figure 9: Share bound by zoning

Back

Inverted Z_i

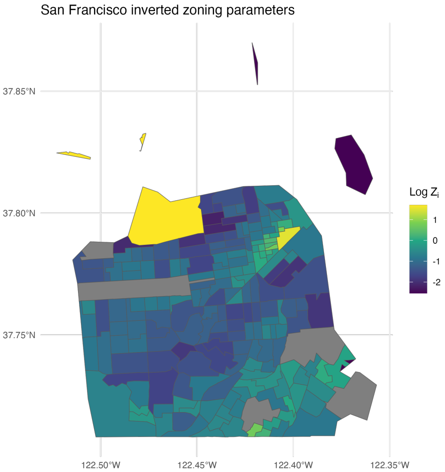

Figure 10: Recovered Z_i

Back

Inverted A_i

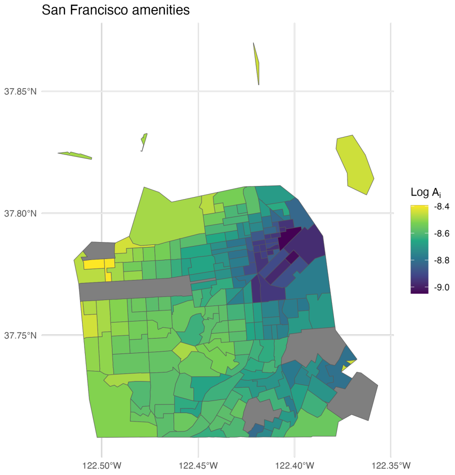

Figure 11: Recovered A_i

Back

Inverted $\mathbb{E}[w_i]$

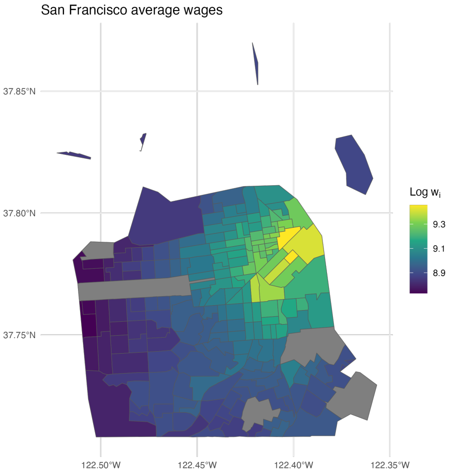

Figure 12: Recovered $\mathbb{E}[w_i]$

Back