The economic geography of climate risk

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Abstract

Projected temperature changes are variable in both their magnitude and geography. This paper studies how nonlinear damages and general equilibrium forces filter this climate risk across time and space. To do so, we build a tractable dynamic spatial model linking countries through trade and migration, and derive analytical first- and second-order welfare approximations that decompose the mean and variance of welfare changes into damage function, trade, and migration components. Using an ensemble of temperature projections from CMIP-6 and an empirically estimated damage function, we show that climate change-induced welfare risk is large. The standard deviation of country-level projected welfare loss across temperature projections is on average over 8% – compared to an average welfare loss of 13% – and is spatially unequal. Climate risk inequality is half as large as global income inequality. We show that spatial linkages reshape not only the level of climate damages, but also the spatial distribution of climate risk: accounting for trade and migration can reduce the standard deviation of welfare changes by up to 40% in low-income, internationally integrated nations that can diversify their exposure to local shocks.

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1 Introduction

There is substantial uncertainty in both the path of global warming over the next century and its spatial distribution.[1] In a high-emissions scenario, state-of-the-art general circulation models (GCMs) from the CMIP-6 ensemble predict between 2 to 5.25°C of warming over populated areas depending on the model used, relative to 2019 (Figure 1, Panel A). However, temperature projections vary widely across regions, with some countries facing a much broader range of projected temperature outcomes across climate models than others (Figure 1, Panels B-D). In this paper, we study how nonlinear damages and the forces of economic geography filter this spatially uneven climate risk, and how accounting for climate risk affects the precision and mean estimates of the economic cost of climate change.

Statistical and economic models of climate damages imply nonlinear relationships between climate shocks and economic outcomes. Statistical models introduce damages that are nonlinear in weather shocks by estimating dynamically persistent state-dependent marginal effects in an environment where the state–average temperature–is itself time-varying. Economic models of climate change imply damages have nonlinear effects as agents are able to adapt to climate change. Because of these nonlinearities, differences in both the magnitude and variability of projected climate change can substantially affect the variance and point estimate of expected welfare. Moreover, since the extent of climate risk differs sharply across regions, accurately evaluating the welfare effects of climate change requires accounting for both the spatial distribution of climate risk and its spatiotemporal propagation. Regions have the potential to adapt to changes in climate through trade and migration, which connect them through the global goods and labor markets and shape how local shocks propagate across space.

Existing literature has made substantial progress in understanding how these adaptation forces shape the cost of climate change (for reviews, see, e.g., Desmet and Rossi-Hansberg, 2024; Balboni and Shapiro, 2025). Dynamic spatial models show that trade and migration allow economies to adapt by reallocating activity toward cooler or more productive regions, while also propagating local shocks through goods and labor markets. We study how these forces shape climate *risk*: the variance of welfare outcomes under different climate

[1] As emphasized by Knight (1921), uncertainty describes situations of ambiguity or ignorance about those probabilities themselves. In practice, we treat this intermodel uncertainty as *risk*: randomness with a known or estimable probability distribution. Our analysis concerns climate risk—the welfare implications of stochastic temperature shocks with empirically estimated distributions derived from global circulation models—rather than Knightian uncertainty over the true climate process.

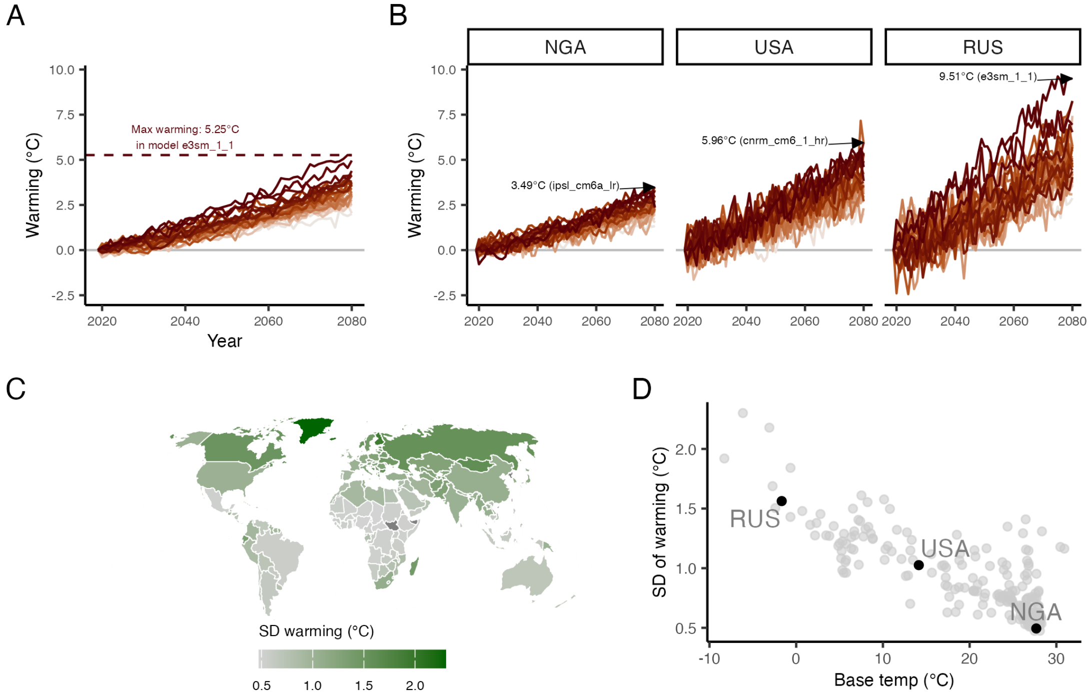

Figure 1: Projected warming across climate models.
Panel A shows the change in global population-weighted annual average near-surface temperature since 2019 across CMIP6 models under the SSP5–8.5 scenario. Panel B reports the corresponding changes in annual average temperature for selected countries. Panel C maps the spatial distribution of the standard deviation in projected warming by 2080 across CMIP models. Panel D plots the standard deviation of country-level warming across models as a function of baseline temperature, showing that colder countries tend to have greater variability in projected warming.

projections. We ask two questions. First, how do global adaptation forces—trade and migration—affect the variance of welfare outcomes due to climate risk, and how do these mechanisms vary across countries? Second, how do nonlinear damages and their spatial propagation influence the expected welfare costs of climate change, relative projections that evaluate welfare given a mean forecast of future temperature?

We find that general equilibrium forces have large and heterogeneous effects on welfare risk. In our simulations, the poorest and most globally integrated economies experience the largest gains from incorporating these adaptation margins. Trade and migration reduce the standard deviation in welfare changes by 2080 (under the SPS5-8.5 emissions trajectory) by as much as 40 percent. Access to foreign markets and migration opportunities allows these economies to diversify exposure to local climate shocks. By contrast, in relatively more closed economies, where domestic conditions dominate welfare fluctuations, trade and migration play a much smaller stabilizing role; instead, risk amplification and attenuation through the damage function dominate. This pattern mirrors their effect on average welfare costs: general equilibrium forces do not amplify damages but rather buffer them, allowing smaller economies to better absorb climate shocks through adaptation and reallocation. Together, these results show that global integration not only mitigates the expected cost of climate change but also redistributes and dampens the associated risk, generating substantial spatial heterogeneity in both the level and volatility of welfare losses.

A thorough quantitative analysis of the questions posed in this paper requires contributions along two key dimensions. First, we estimate a dynamic, non-linear damage function that maps changes in temperature into changes in model-implied productivity. This function anchors the link between physical climate shocks and the economic fundamentals of our model. We construct an annual panel of country-level data combining observed temperature realizations with model-implied productivity measures, and use it to estimate the causal relationships between temperature fluctuations and economic productivity. Following the climate impacts literature, we employ a flexible functional form that allows for non-linear and dynamic responses (Burke et al., 2015), building on recent approaches that trace the delayed effects of climate shocks on economic performance (Nath et al., 2024; Bilal and Känzig, 2024). Specifically, we incorporate distributed lags of temperature to capture adjustment dynamics over the subsequent decade. Because we estimate this relationship on a long panel, we are able to disentangle persistent effects weather shocks, while controlling for global trends and country fixed effects. This produces a rich empirical mapping from temperature to productivity that can be embedded directly into our quantitative model. Combining this estimated damage function with stochastic temperature projections from CMIP climate models enables us to simulate country-specific productivity distributions across temperature projections, and to propagate productivity shocks through the model's equilibrium to compute real income changes.

Second, we develop a tractable dynamic spatial model that links countries through trade and migration, which act as both margins of adaptation—allowing agents to adjust to local productivity shocks—and as mechanisms of propagation, transmitting weather shocks across space (via trade) and time (via migration). The model captures the key theoretical forces of adaptation and propagation while remaining analytically tractable and computationally efficient, allowing us to simulate numerous stochastic climate realizations and trace their welfare implications globally.

Within this environment, we derive a first-order welfare approximation (reminiscent of the results in Kleinman et al., 2024a; Kleinman et al., 2024b) for welfare changes induced by small productivity shocks that incorporates general equilibrium feedback, decomposed according to the contribution of direct impacts (i.e., through the damage function) versus those coming through trade and migration. We then extend this to a second-order approximation that captures curvature in the welfare function arising from nonlinear damages and higher-order GE effects. These results yield a set of transparent analytical decompositions that separate welfare changes into direct, trade, and migration components—each of which can be computed empirically and compared across countries.

This framework allows us to trace how the impact of climate shocks accumulates over time and how adaptation and propagation interact to shape the distribution of welfare risk. Using country-specific temperature projections from the CMIP ensemble, we simulate global welfare trajectories and decompose them into their underlying channels. Together, these elements provide a unified, empirically grounded framework that connects climate projection variability to the geography of welfare risk.

We then use our framework to decompose both the expected cost and the variance of welfare into direct and general equilibrium (GE) components for every country in the world. GE forces have highly heterogeneous implications for welfare. In much of sub-Saharan Africa and South and Southeast Asia, trade and migration attenuate direct welfare losses by reallocating resources toward less-affected regions. In other areas, GE forces can instead exacerbate losses through adverse terms-of-trade effects—when local productivity declines relative to trading partners—or through migration inflows and outflows that redistribute exposure to shocks. These contrasting channels generate substantial spatial heterogeneity in the global geography of climate costs.

When we turn to welfare variance, we find that GE forces dampen volatility nearly everywhere, but with sharp cross-country differences. Countries that are highly integrated in global trade experience the largest stabilizing gains, with welfare variance falling by up to 25 percent in some smaller economies such as Chad and Greece. In contrast, for larger economies such as the United States or Russia, GE channels have little effect on welfare volatility because domestic shocks dominate. These results carry clear policy implications: openness to trade and migration provides a form of insurance against local climate shocks, helping to stabilize welfare under climate risk.

Related literature A large body of research has used reduced-form estimates of the causal relationship between temperature and GDP to project the global economic consequences of climate change (see Hsiang, 2025; Lemoine et al., 2025, for recent reviews). This literature consistently finds a non-linear relationship between temperature and output (Burke et al., 2015; Kalkuhl and Wenz, 2020; Nath et al., 2024): warming tends to raise productivity in cold countries but reduce it in hot ones, with most studies identifying an optimal average temperature of around 14°C. Studies differ on persistence: while Burke et al. (2015) link temperature to GDP growth—implying compounding long-run effects—others argue that only changes in temperature affect growth, with impacts that can persist for a decade or more (Nath et al., 2024; Bilal and Känzig, 2024). In this paper, we adopt an estimation strategy that accommodates both non-linear impacts and persistent—but not permanent—effects of a sustained change in temperature.

A central challenge in this literature is the large variance surrounding projected economic damages from climate change (Newell et al., 2021). For instance, Burke et al. (2015) report a 95% confidence interval for global GDP losses by 2100 that ranges from –60% to +50%. Importantly, this wide range reflects only statistical uncertainty in the estimated damage function, neglecting the role of climate risk. Subsequent work—often outside the core GDP–climate literature—has expanded this analysis to include uncertainty about the future climate, typically by bootstrapping damage estimates across ensembles of climate model realizations. For example, Carleton et al. (2022) account for both damage-function and climate risk in their projections of climate-induced mortality, finding that the two sources contribute roughly equally to variance in 2100 outcomes. Similarly, Burke et al. (2023) show that uncertainty in future climate trajectories adds a comparable magnitude of uncertainty to Social Cost of Carbon calculations to that arising from estimation of the damage function. A smaller emerging literature decomposes other sources of uncertainty in projected impacts: Schwarzwald and Lenssen (2022) highlights the role of internal climate variability—differences across realizations within a single climate model—which we do not explicitly analyze in this paper.

A key limitation of these reduced form studies is that they treat economies as spatially isolated, neglecting how economic geography forces both create spatial spillovers but also the ability for local adaptation through trade and migration. As GDP is itself an endogenous outcome influenced by cross-border interactions, omitting these spatial linkages may overstate projected welfare damages and their variance, and obscure how global adaptation could redistribute and reduce climate risk (Zappala, 2024; Bilal and Känzig, 2024).

This paper contributes to the growing literature on how trade and migration shape the welfare consequences of climate change. Recent research has expanded classic integrated assessment models (IAMs) to capture spatial heterogeneity, trade, and migration, giving rise to spatial integrated assessment models (S-IAMs) that examine how geography influences both the level and distribution of climate damages. Desmet and Rossi-Hansberg (2015) formalized dynamic spatial equilibrium frameworks showing that spatial frictions—especially in trade and migration—critically determine aggregate and local climate costs. Subsequent research has broadened this agenda along several dimensions. Desmet et al. (2021) evaluate probabilistic projections of local sea-level rise in a dynamic spatial model. Conte et al. (2021) study how warming reshapes global patterns of industrial specialization, while Cruz and Rossi-Hansberg (2024) develop a global S-IAM with endogenous migration, innovation, and demographic dynamics. Balboni (2025) and Rudik et al. (2022) analyze temperature and flooding impacts in models with forward-looking migration, and Krusell and Smith Jr (2022) emphasize forward-looking investment and savings as alternative adaptation channels. Bilal and Rossi-Hansberg (2023) highlight how anticipation of extreme events, spatial capital adjustment, and externalities affect long-run outcomes. Dingel and Meng (2025) show that spatially correlated temperature shocks can amplify inequality by altering the gains from trade. The interaction between environmental policy and geography has also been explored (Acemoglu et al., 2016; Conte et al., Forthcoming).

Despite these advances, most dynamic spatial analyses evaluate deterministic climate scenarios, abstracting from variability across temperature paths. A notable exception is Desmet et al. (2021), who examine heterogeneous flooding risks under alternative Representative Concentration Pathways (RCPs). In contrast, we incorporate stochastic temperature dynamics into a spatial general equilibrium framework with trade and migration. Importantly, we derive analytical first- and second-order welfare decompositions that isolate how both the estimated damage function and general equilibrium forces shape both the level and variance of welfare under climate risk across time and space.

Finally, this paper is also related to recent work that develops decompositions of welfare exposure to shocks in quantitative spatial models. Kleinman et al. (2024a) derive 'friend–enemy' matrices that characterize the first-order elasticity of real income to productivity changes across countries in a trade model. Kleinman et al. (2024b) and Donald et al. (2025) extend these sufficient statistics to spatial settings with migration. We build on this linear-exposure logic but go further by deriving both first- and second-order welfare approximations that quantify not only the expected welfare cost but also the variance and curvature components arising from adaptation and propagation. This allows us to isolate how trade and migration separately shape both the mean and dispersion of welfare impacts across regions.

The rest of this paper is organized as follows. In Section 2 we describe our dynamic spatial equilibrium model, which allows us to characterize how productivity shocks caused by changes in temperature propagate across space due to trade and migration. In Section 3 we develop a framework to decompose how projected welfare, and its uncertainty, depends on direct productivity damages, trade, and migration. In Section 4 we describe how we estimate model parameters. In Section 5, we present our results. We show projections and decompositions of the welfare impacts of climate change, and its variance. Finally, in Section 5, we conclude. We relegate many of the model and quantification details to the Appendix.

2 Model

In this section, we develop a simple dynamic spatial equilibrium model in which countries are linked through trade and migration. The model provides a tractable framework to study how shocks to fundamentals propagate across time and space through these two margins of adjustment. The environment extends the canonical Armington (Anderson, 1979) structure to a dynamic setting in which agents' migration decisions generate intertemporal spillovers. The world is populated by measure $\bar{L}$ workers dispersed across all countries, each of whom is endowed with one unit of labor that they supply inelastically to their nation's final goods firm. There are N countries indexed by $o, d \in \{1, \ldots, N\}$. Time is discrete and indexed by t. The timing within each period is as follows:

1. Workers 'wake up' in origin country o at the beginning of period t, and make a one-period ahead migration decision to destination country d.
2. Populations update according to realized migration decisions.
3. Production, trade, and consumption take place in each destination.

Workers migrate across regions paying a bilateral migration cost μ_{od} expressed in terms of utility. Trade is costly and subject to standard iceberg trading costs τ_{od}, and there is no ability for agents to save.

2.1 Workers

Utility and Consumption. Conditional on being in a country d, workers supply one unit of labor inelastically to the country's representative firm and earn wage $w_{d,t}$. They consume a basket of goods sourced from nations o, and have CES preferences over national varieties, and enjoy country-specific amenities $A_{d,t}$. The value of residing in country d is given by

$$V_{d,t} = \max_{\{C_{od,t}\}} A_{d,t} \left(\sum_o C_{od,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad \text{subject to } \sum_o p_{od,t} C_{od,t} \leq w_{d,t},$$

where $p_{od,t} = \tau_{od} p_{o,t}$ is the price of the variety from country o adjusted for the iceberg trading cost between countries. Expenditure minimization implies the standard CES price index

$$P_{d,t} = \left(\sum_o p_{od,t}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

Demand for each imported variety can be written as

$$C_{od,t} = \left(\frac{p_{od,t}}{P_{d,t}} \right)^{-\sigma} \frac{w_{d,t}}{P_{d,t}}. \tag{1}$$

Migration. In each period t, there is a continuum of workers indexed by ω residing in origin o of measure $L_{o,t-1}$. Each worker decides where to live and work in period t. Workers draw idiosyncratic preference shocks $\epsilon_{od}(\omega)$ that are distributed Fréchet $(1, \varepsilon)$ iid across countries. Each worker solves

$$\max_d \left\{ \frac{V_{dt}}{\mu_{od}} \epsilon_{odt}(\omega) \right\}, \tag{2}$$

where μ_{od} captures bilateral migration costs of moving from o to d. The resulting migration probabilities of moving from o to d satisfy

$$M_{od,t} = \frac{(V_{d,t}/\mu_{od})^{\varepsilon}}{\sum_{d'} (V_{d',t}/\mu_{od'})^{\varepsilon}}, \tag{3}$$

and ex–ante expected welfare for workers in o is

$$\mathcal{W}_{ot} \propto \left(\sum_d (V_{dt}/\mu_{od})^{\varepsilon} \right)^{1/\varepsilon}. \tag{4}$$

2.2 Technology and Firms

Each country produces a differentiated Armington variety using labor as the only factor of production. The representative firm in country d has linear technology using productivity $Z_{d,t}$,

$$Y_{d,t} = Z_{d,t} L_{d,t}.$$

Perfectly competitive firms take wages $w_{d,t}$ and prices $p_{d,t}$ as given, and solve

$$\max_{L_{d,t}} \; p_{d,t} Z_{d,t} L_{d,t} - w_{d,t} L_{d,t}.$$

Since labor is the only factor, wages equal the value of marginal product such that

$$w_{d,t} = p_{d,t} Z_{d,t}. \tag{5}$$

2.3 Market Clearing

Output market clearing requires that total production equals global demand for each variety such that

$$Y_{o,t} = \sum_d \tau_{od} C_{od,t} L_{d,t}. \tag{6}$$

The aggregate labor market must also clear. The initial population distribution is such that $\sum_o L_{o,0} = \bar{L}$, and there is no population growth. Given migration probabilities in (3), the population in each country evolves according to

$$L_{d,t} = \sum_o M_{od,t} L_{o,t-1}. \tag{7}$$

2.4 Equilibrium

We are now ready to define the notion of equilibrium. We first define a period-t specific equilibrium, before defining a dynamic equilibrium.

Definition 2.1 (Period-t Equilibrium)**.** Given fundamentals $\{A_{o,t}, Z_{o,t}\}$, elasticities (ε, σ), iceberg trade costs $\{\tau_{od}\}$, and initial populations $\{L_{o,t-1}\}$, a period-t equilibrium is a set of wages $\{w_{o,t}\}$, prices $\{p_{o,t}\}$, consumption $c_{od,t}$, and population distribution $\{L_{o,t}\}$ such that:

1. Workers choose consumption optimally, satisfying equation (1).
2. Firms choose labor inputs to maximize profits, and wages satisfy equation (5).
3. Goods markets clear, satisfying equation (6).
4. The population distribution across countries satisfies equation (7).

Definition 2.2 (Dynamic Equilibrium)**.** Given sequences of fundamentals $\{A_{o,t}, Z_{o,t}\}_{t=0}^{\infty}$ and an initial population distribution $\{L_{o,0}\}$, a dynamic equilibrium is a sequence $\{p_{o,t}, w_{o,t}, L_{o,t}\}_{t=0}^{\infty}$ such that each period t constitutes a period-t equilibrium and the population evolves according to (7).

A stationary equilibrium corresponds to a population distribution in which $L_{o,t} = L_{o,t-1}$ for all i.[2]

2.5 Discussion

The model captures how spatial adjustment to shocks unfolds through contemporaneous trade and migration decisions. Each period's allocation reflects current fundamentals: workers respond to observed wages, prices, and amenities. This static behavior generates a dynamic sequence of equilibria through the law of motion for populations—today's realized distribution of workers $\{L_{o,t}\}$ becomes tomorrow's inherited state.

Under this formulation, persistence in spatial outcomes arises from the gradual reallocation of workers across space. Shocks to fundamentals—such as changes in productivity, amenities, or climate—affect wages and prices contemporaneously, leading to new migration flows that redefine the distribution of population in

[2]That is, the equilibrium population vector is proportional to the dominant eigenvector of the migration matrix $\mathbf{M}_t$.

subsequent periods. Trade linkages transmit these shocks across locations within a period, while migration links outcomes across periods.

3 Welfare Decompositions

This section summarizes how trade and migration shape the propagation of shocks into welfare. We build four components: (i) a first–order welfare decomposition for productivity shocks; (ii) an analogous result for (initial) population shocks; (iii) a dynamic accumulation result that organizes propagation over time; and (iv) variance and Jensen (curvature) decompositions that map uncertainty in fundamentals into uncertainty in welfare.

We let $\mathcal{W}_{o,t}$ denote *ex ante* welfare for agents starting period t in country o. This is the welfare for agents before they receive idiosyncratic location specific preference shocks. Defining indirect utility as $V_{o,t} = A_{o,t}\frac{w_{o,t}}{P_{o,t}}\ \forall o$, (i.e, amenity-adjusted real income), country level welfare $\mathcal{W}_{o,t}$ takes the following form:[3]

$$\mathcal{W}_{o,t} \;=\; V_{o,t}\left(\sum_{d}\mu_{od}^{-\varepsilon}\left(\frac{V_{d,t}}{V_{o,t}}\right)^{\varepsilon}\right)^{1/\varepsilon}. \tag{9}$$

3.1 First–order welfare impact of productivity shocks

We consider the effect, within period t, of a small productivity shocks $d\ln Z_t$ and initial population shocks $d\ln L_{t-1}$ on period-t welfare for any country. Absent productivity shocks, there is mechanical convergence to the steady state so that $d\ln L_{t-1}$ is nonzero.

The following proposition decomposes the first order changes to welfare as a consequence to shocks in productivity and starting population.

Proposition 1 (First–order decomposition)**.** In any period-t equilibrium, the first-order change in the log

[3]When we discuss global welfare in any period t, we adopt a utilitarian aggregator. Namely,

$$\Upsilon_t \;=\; \sum_{o} L_{o,t}\,\mathcal{W}_{o,t}, \tag{8}$$

welfare for any country $o \in \{1, \cdots, N\}$ can be written as,

$$d \ln \mathcal{W}_t = (\mathbf{I} + \boldsymbol{\mathcal{T}}_{t-1} + \boldsymbol{\mathcal{M}}_{t-1})\, d \ln Z_t + \mathcal{L}_t. \tag{10}$$

where $d \ln Z_t$ is an N-dimensional vector of productivity shocks.

Proof. See Appendix (B.1). □

We refer to $\mathbf{I}\, d \ln Z_t$ as the direct, or damage function effect, $\boldsymbol{\mathcal{T}}_{t-1}\, d \ln Z$ as the trade effect, and $\boldsymbol{\mathcal{M}}_{t-1}\, d \ln Z_t + \mathcal{L}_t$ as the migration effect. These matrices are functions of trade, income, and migration shares. Dropping time subscripting, the trade effect is given by,

$$\boldsymbol{\mathcal{T}} = (\mathbf{I} - \mathbf{T}^\top)\mathbf{A}^Z$$

while the migration multiplier on the shock is,

$$\boldsymbol{\mathcal{M}} = \underbrace{(\mathbf{M} - \mathbf{I})\mathbf{A}^Z}_{\text{migration option}} + \underbrace{(\mathbf{M} - \mathbf{I})(\mathbf{I} - \mathbf{T}^\top)\mathbf{A}^Z}_{\text{trade-migration interaction}}$$

where the matrices $\mathbf{T}$ is the trade share matrix, and $\mathbf{M}$ is the migration matrices; i.e.,

$$\mathbf{T}_{od} = \left(\frac{\tau_{od}\, p_o}{P_d}\right)^{1-\sigma}, \qquad \mathbf{M}_{od} = \left(\frac{V_d/\mu_{od}}{\mathcal{W}_o}\right)^{\varepsilon},$$

and $\mathbf{A}^Z$ maps productivity shocks into prices changes ($d \ln p = \mathbf{A}^Z\, d \ln Z$). Further,

$$\mathcal{L}_t = \mathbf{M}(I - \mathbf{T}^\top)\mathbf{A}^L\, d \ln L,$$

where $\mathbf{A}^L$ maps initial-population shocks into price changes ($d \ln p = \mathbf{A}^L\, d \ln L^0$). Both $\mathbf{A}$-operators depend on primitives and statistics of the state (the spatial distribution of population and productivity).[4] The

[4] The relevant statistics of the state are given by the matrices $(\mathbf{L}, \mathbf{M}, \mathbf{T}, \mathbf{S})$. Defining,

$$\mathbf{Q} = \left(\sigma \mathbf{I} - \mathbf{S} - (\sigma - 1)\mathbf{S}\mathbf{T}^\top\right)^{-1}(\mathbf{S} - \mathbf{I})$$

to be the pass-through matrix of endowment shocks to prices, where $\mathbf{S}_{od} = \mathbf{T}_{od}\frac{w_d L_d}{w_o L_o}$, the price effect in our model with migration decomposition illustrates how shocks to fundamental productivity interact with the forces of economic geography to shape welfare outcomes.

The **direct effect**, $d \ln Z_t$ is read off the damage function. A local productivity shock immediately changes real income in the affected region, holding prices and migration patterns fixed. The **trade effect** captures how terms of trade adjust to the full vector of productivity shocks. Through international trade, each region's welfare depends its productivity relative to those of its trading partners. All else equal, negative productivity shocks can improve a nation's terms-of-trade as the price of its export appreciates, changing real consumption possibilities. This is the mechanism through which trade acts as a form of adaptation. Quantitatively, the strength of this adaptation mechanism depends on how a nation's productivity changes relative to those its trading partners.

The term captures the **migration option effect**. As agents can relocate across regions, a productivity shock elsewhere changes the relative attractiveness of destinations. Agents who stay enjoy real income changes from emigration, and a positively selected on their preference shock. For residents of origin o, higher productivity in d increases the option value of migrating there, even if they ultimately stay, since equilibrium migration probabilities M_{od} depend on relative utilities across all destinations. The fourth term represents the **interaction effect between trade and migration**. Conditional on migration responses, changes in productivity elsewhere alter both where individuals move and how potential migration destinations' terms-of-trade adjust. Thus, the gains from mobility depend not only on the dispersion of productivities but also on the geography of trade costs and price adjustment across destinations. Finally, the term $\mathcal{L}_t$ in the decomposition captures welfare gains from the population distribution converging towards the steady state.

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is,

$$\mathbf{A}^Z = \left(\mathbf{I} - \mathbf{Q}\epsilon(\mathbf{I} - \mathbf{L}^\top \mathbf{M})(\mathbf{I} - \mathbf{T}^\top)\right)^{-1} \mathbf{Q} \left(\mathbf{I} + \epsilon(\mathbf{I} - \mathbf{L}^\top \mathbf{M})\right)$$

and prices respond to population shocks according to,

$$\mathbf{A}^L = (\mathbf{I} - \mathbf{Q})^{-1} \mathbf{Q} \mathbf{L}^\top$$

where $\mathbf{L}_{od}$ is the share of persons in destination d who originated in o.

Using this decomposition, we can trace how the effects of productivity shocks accumulate over time.

$$\begin{aligned}\ln \mathcal{W}_t - \ln \mathcal{W}_0 &\approx \sum_{n=1}^{t} d\ln \mathcal{W}_n \\ &= \sum_{n=1}^{t} \left(\mathbf{I} + \boldsymbol{\mathcal{T}}_{n-1} + \boldsymbol{\mathcal{M}}_{n-1}\right) d\ln Z_n + \mathcal{L}_t,\end{aligned}$$

where $\boldsymbol{\mathcal{T}}_{n-1}$ captures the propagation of shocks through trade at time n, $\boldsymbol{\mathcal{M}}_{n-1}$ captures migration and migration–trade interaction effects, and $\mathcal{L}_t$ summarizes the population convergence effect.

This expression emphasizes that the total welfare change at any date reflects the cumulative history of productivity shocks weighted by the structure of trade and migration linkages in previous periods. Trade linkages propagate shocks contemporaneously across space, while migration linkages propagate them intertemporally across periods. The term $\mathcal{L}_t$ captures additional adjustment arising from gradual reallocation of population, which can either amplify or dampen past shocks depending on how migration reshapes exposure to productivity across space.

3.2 Welfare curvature: second order effects

We now investigate the curvature of the welfare function, which shapes how general equilibrium forces can amplify or attenuate climate risk. Moreover, the second derivative of the welfare function informs us about the magnitude of the correction we must make to the expected welfare loss when integrating over the distribution of future climate, instead of evaluating welfare along the mean trajectory.

Within a given period t, the second–order response of welfare to productivity shocks $d\ln Z_t$ is given by the following proposition.

Proposition 2 (Second–order welfare decomposition)**.** In any period–t equilibrium, the second–order change in log welfare for any country $o \in \{1, \ldots, N\}$ is

$$d^2 \ln \mathcal{W}_o = \underbrace{(\sigma - 1)\, \mathrm{Var}_{\mathbf{T}_{\cdot d}}(d \ln p_o)}_{\text{trade adaptation}} + \underbrace{\varepsilon\, \mathrm{Var}_{\mathbf{M}_{o\cdot}}(d \ln V_d/V_o)}_{\text{migration adaptation}}$$
$$+ \underbrace{\sum_{od} M_{od}\big(d^2 \ln V_d/V_o\big) - \sum_{o} \mathbf{T}_{od}\big(d^2 \ln p_o/p_d\big)}_{\text{second–order price effects}} .$$

Proof. See Appendix B.4. □

Standard spatial models often evaluate welfare at the expected path of fundamentals, computing $\ln \mathcal{W}(\mathbb{E}[Z_t])$. However, when welfare is nonlinear in productivity—as is the case once trade and migration linkages create convexities—the expected welfare loss across climate realizations, $\mathbb{E}[\ln \mathcal{W}_t]$, can differ substantially from the welfare computed at expected fundamentals. This distinction follows from Jensen's inequality, which states that for any function $f(\cdot)$ and random variable x, $f(\mathbb{E}[x]) \gtrless \mathbb{E}[f(x)]$ depending on whether $f(\cdot)$ is either convex or concave. Applied to welfare, this implies that the bias from failing to account for climate risk depends on,

$$\mathbb{E}_k \left[d \ln \mathcal{W}_t^{(k)}\right] - d \ln \mathcal{W}_t \left(\mathbb{E}_k[d \ln Z_t]\right) \approx \frac{1}{2} d^2 \ln \mathcal{W}_t \times \mathrm{Var}\left(d \ln Z_t^{(k)}\right)$$

depends on whether welfare is locally convex or concave in productivity. In our context, we are interested in what drives the difference between these two objects. We can use the second–order decomposition of welfare to characterize the shape of this relationship.

The second–order terms capture how dispersion in local prices and migration returns translates into curvature in welfare. **Trade adaptation** makes the welfare function more convex: when productivity shocks induce price dispersion across trading partners, trade allows countries to substitute toward cheaper imports, raising welfare through within–period reallocation. A higher elasticity of substitution (σ) amplifies this effect, reflecting stronger risk–sharing and a greater capacity to smooth localized shocks. **Migration adaptation** parallels this mechanism across space. When migration frictions are low (high ε), workers can reallocate toward destinations where real incomes rise, yielding welfare gains from the cross–sectional variance in destination–specific returns.

Taken together, trade and migration adaptation make the welfare function more nonlinear in fundamentals: by linking regions through prices and mobility, they transform local shocks into convex welfare responses. Finally, the residual term captures higher–order general–equilibrium effects that reflect feedback from expenditure and labor reallocation into local prices. Together, these components quantify how the economy's spatial margins of adjustment shape not only the mean but also the volatility and curvature of welfare responses to shocks.

3.3 Mean effects and variance decomposition

This framework allows us to decompose the sources of welfare change into their underlying propagation channels. For a sequence of shocks $\{d\ln Z_t^{(k)}\}$—along a certain simulated climate or productivity paths (k)—the contribution of each channel to cumulative welfare changes can be written as:

$$\begin{aligned}
\text{Direct role}_t^{(k)} &= \frac{\sum_{n=1}^{t} d\ln Z_n^{(k)}}{\ln \mathcal{W}_t - \ln \mathcal{W}_0},\\
\text{Trade role}_t^{(k)} &= \frac{\sum_{n=1}^{t} \boldsymbol{\mathcal{T}}_{n-1} d\ln Z_n^{(k)}}{\ln \mathcal{W}_t - \ln \mathcal{W}_0},\\
\text{Migration role}_t^{(k)} &= \frac{\sum_{n=1}^{t} \boldsymbol{\mathcal{M}}_{n-1} d\ln Z_n^{(k)} + \hat{\mathcal{L}}_t}{\ln \mathcal{W}_t - \ln \mathcal{W}_0}.
\end{aligned}$$

These expressions quantify the relative importance of each mechanism in shaping cumulative welfare changes. The direct term isolates the effect of own-country productivity shocks. The trade term measures how exposure through import and export linkages transmits foreign shocks. The migration term captures how the reallocation of workers, together with induced changes in population weights and option values (the $\hat{\mathcal{L}}_t$ component), contributes to global adjustment.

Having established how trade and migration shape the mean response of welfare to shocks, we now examine how they affect the *variance* of welfare across climate realizations (k). For each time t, we decompose the contribution of direct, trade, and migration channels to the cross–chain variance of welfare as

$$
\begin{aligned}
\text{Direct Share}_t &= \frac{\mathrm{Var}_k\left(\sum_{n=1}^{t} d\ln Z_n^{(k)}\right)}{\mathrm{Var}_k(\ln \mathcal{W}_t - \ln \mathcal{W}_0)}, \\
\text{Trade Share}_t &= \frac{\mathrm{Var}_k\left(\sum_{n=1}^{t}(\mathbf{I}+\boldsymbol{\mathcal{T}}_{n-1}^{(k)})\, d\ln Z_n^{(k)}\right) - \mathrm{Var}_k\left(\sum_{n=1}^{t} d\ln Z_n^{(k)}\right)}{\mathrm{Var}_k(\ln \mathcal{W}_t - \ln \mathcal{W}_0)}, \\
\text{Migration Share}_t &= \frac{\mathrm{Var}_k\left(\sum_{n=1}^{t}(\mathbf{I}+\boldsymbol{\mathcal{M}}_{n-1}^{(k)})\, d\ln Z_n^{(k)}\right) - \mathrm{Var}_k\left(\sum_{n=1}^{t} d\ln Z_n^{(k)}\right)}{\mathrm{Var}_k(\ln \mathcal{W}_t - \ln \mathcal{W}_0)}.
\end{aligned}
$$

The residual share captures interaction effects and the covariance structure arising from serial and spatial autocorrelations.

The **direct share** measures the portion of welfare variance that would arise in the absence of any trade or migration responses—that is, the propagation of uncertainty purely through shocks to local productivity. In isolation, this accounts for all variance in a world of autarky and immobile labor.

Trade and migration, by contrast, reshape how uncertainty transmits across space. Trade connects countries through their price indices: if a volatile region trades with more stable partners, international trade smooths welfare by diversifying consumption baskets and dampening local volatility. Conversely, when a stable region trades extensively with highly uncertain partners, it imports volatility through fluctuations in foreign prices.

Migration operates analogously through the reallocation of workers. When migration frictions are low, workers can relocate away from volatile or adversely affected regions toward more stable ones, attenuating welfare dispersion. However, when migration primarily links countries with correlated or volatile fundamentals, mobility can instead transmit instability across borders.

As such, whether the forces of economic geography in trade and migration amplify or attenuate welfare variance is ultimately an empirical question.

4 Estimation

4.1 Data

GDP data We obtain estimates of GDP per capita (GDP-PC) from the World Bank's *World Development indicators* (WDI) database, expressed in constant 2015 US dollars. These data have been widely used in studies that project the GDP consequences of climate change (e.g., Burke et al., 2015; Nath et al., 2024). Our dataset forms an unbalanced panel covering the period 1960–2019; we exclude observations beyond 2019 to avoid distortions associated with the COVID-19 pandemic. Among the 199 countries in the dataset, 87 have complete GDP-PC coverage for the entire 1960–2019 period, while the remaining countries exhibit missing values in early years (see Appendix Figure A1 for details on data availability by country). The WDI also provides us with country-year level population estimates.

Historical temperatures We match these GDP data with temperature data obtained from the ERA5 dataset (Muñoz-Sabater et al., 2021). ERA5 provides daily, grid-cell level temperature and precipitation estimates, which we population weight using gridded population estimates, before aggregating to the country-year level.[5]

Trade and migration data We use bilateral trade flows, expressed in nominal dollars, for years 2000-2016 from the U.S. Gravity Portal (Gurevich and Herman, 2018). For historical bilateral migration flows, we rely on those from Abel and Cohen (2022), who use demographic accounting methods to recover bilateral migration flows in 1990,1995,2005, and 2010.

CMIP projections We use 35 CMIP-6 projections for the RCP 8.5 forcing scenario made available through the Copernicus website.[6] We aggregate CMIP-6 projections to the country level by population-weighting gridcell level estimates.[7]

---

[5]Populations weights are time invariant, and calculated using the Gridded Population fo the World dataset, available here: `https://sedac.ciesin.columbia.edu/data/collection/gpw-v4`

[6]See: https://cds.climate.copernicus.eu/datasets/projections-cmip6.

[7]We note that CMIPs reflect 'ensembles of opportunity' (Rasmussen et al., 2016): they are sampled from participating GCMs and do not align with less granular forecasts like MAGICC (Model for the Assessment of Greenhouse Gas Induced Climate Change, a 'simple' GCM). One approach to align granular CMIP uncertainty with estimates of aggregate uncertainty is to develop surrogate models by resampling from the CMIP distribution (as in Carleton et al., 2022; Tran-Anh and Ngo-Duc, 2024) or estimating and drawing from the CMIP DGP, which we aim to do in future work.

4.2 Model estimation and calibration

We set $\sigma - 1 = 4$ following Simonovska and Waugh (2014) and set $\epsilon = 2$ following Cruz and Rossi-Hansberg (2024).

Estimating spatial frictions To estimate τ_{od} and μ_{od}, we follow the 'covariates-based approach' of Dingel and Tintelnot (2025). That is, we parameterize our spatial frictions as a function of covariates. We do this to avoid overfitting estimates of τ_{od} and μ_{od} on noisy trade and migration flows Many trade and migration flows in the data are zero. Subsequently, fitting trade and migration costs to exactly rationalize historical flows as a model equilibrium would generate infinite trade or migration costs for some country pairs. By smoothing the data with a low rank approximation to the trade and migration costs, we allow for trade and migration flows to occur in counterfactuals that are not seen in historical data.

To estimate our spatial frictions, we use the model's gravity regressions for trade and migration flows,

$$\begin{aligned} \mathbf{M}_{odt} &= \exp\left(-\epsilon \ln \mu_{odt} + \xi_{ot} + \xi_{dt}\right) \\ \mathbf{T}_{odt} &= \exp\left(-(\sigma - 1) \ln \tau_{odt} + \chi_{ot} + \chi_{dt}\right) \end{aligned} \tag{11}$$

We treat these as estimating equations in which we suppose $\mu_{odt}^{-\epsilon} = \exp(\Gamma X_{od}) u_{odt}$ and $\tau_{odt}^{1-\sigma} = \exp(\Lambda X_{od}) v_{odt}$ where $\mathbb{E}\left[u_{odt} \mid \xi_{ot}, \xi_{dt}, X_{od}\right] = \mathbb{E}\left[v_{odt} \mid \xi_{ot}, \xi_{dt}, X_{od}\right] = 1$. This allows us to exchange $\tau_{odt}^{1-\sigma}$ and $\mu_{odt}^{-\epsilon}$ for $\exp(\Gamma X_{od})$ and $\exp(\Lambda X_{od})$ and estimate Equations (11) using a Poisson pseudo-maximum likelihood estimator with high-dimensional fixed effects (Silva and Tenreyro, 2006; Correia et al., 2020).[8] Inside X_{od} we use the standard gravity regressors: log distance, and indicators for colonial history, legal and linguistic similarities, and contiguity (Gurevich and Herman, 2018). Estimation results are available in Table A1.

Estimating the damage function We invert the model using our GDP and population data to recover Z_{ot}

[8] We enforce $\tau_{oo} = \mu_{oo} = 1$ by including $1(o = d)$ as a regressor in X_{od} and rescaling all elements of $\hat{\Gamma}$ and $\hat{\Lambda}$ accordingly.

for each country-year in the data. [9] To estimate damages, we run regressions of the form,

$$\Delta \log Z_{ot} = \sum_{\ell=0}^{L} \beta_{0,\ell} \Delta C_{o,t-\ell} + \beta_{1,\ell} \Delta (C_{o,t-\ell})^2 + \eta_o + \eta_t + \delta_o \cdot t + e_{ot} \tag{12}$$

where $C_{i,t}$ is temperature and Δ first-differences the data. The term η_t is a year fixed effect, η_o is a country-fixed effect, and δ_o is a country specific quadratic time trend. The model estimates the causal effect of temperature on productivity under the assumption that year-to-year fluctuations in temperature are as good as randomly assigned, conditional on our time controls.

In this model, the impacts of a change in temperature are assumed to unfold over the L years following the change. A large literature has examined the persistence of the impacts of temperature shocks, and we follow here a recent literature (Nath et al., 2024; Bilal and Känzig, 2024) which assumes that the impacts of temperature on growth rates are persistent, but not permanent.[10] We use $L = 10$ lags to capture persistent effects of temperature shocks on productivity.[11]

Estimating the damage function on the model primitives is similar to the estimation strategy in Cruz and Rossi-Hansberg (2024). However, instead of estimating the parameters of a parametric damage function, we allow for flexible, state-dependent, dynamic marginal effects. As we fit our model such that GDP per capita, $y_{ot} = p_{ot} Z_{ot}$, Equation 12 can be thought of as a GDP-per-capita-temperature regression in which we control for spatial spillovers parametrically using the structure of the model, as in Zappala (2024).[12] Appendix Figure A2 compares our estimated productivity damage function to a damage function for raw GDP-PC values, showing that damages estimated on model implied productivity leads to steeper damages.

Figure 2 presents the estimated relationship between temperature and productivity from Equation (12). Panel

[9] Recovering productivity (Z_{ot}) involves solving a system of nonlinear equations to separate out p_o, which is unobserved, from data on GDP per person. Solving this system requires GDP and population data for every country in the trading network in every year. However, some rows in the WDIs are missing. To interpolate missing GDP data in the WDIs, we fit a regression of log GDP per capita on log population, and add country and year fixed effects, as well as country-specific slopes on time and US GDP per capita. To construct the trade matrix historically, we use our estimated trade costs τ_{od}. We then invert the trade gravity equation to recover a panel of Z_{ot}. We flag observations for which GDP per capita data in any country-year were interpolated and drop those observations from the estimation sample. See Appendix C.1 for details.

[10] Allowing for the possibility of permanent growth effects of temperature increase, similar to Burke et al., 2015, would increase our damage estimates

[11] Unlike Bilal and Känzig (2024), we only use local temperature as a regressor, and abstract from the effects of global temperature on local productivity, which are absorbed in η_t, as global temperature shocks are difficult to identify from the limited time-series.

[12] To see this, note that our model implies that prices encode information about trade and migration links across space (see details in Appendix). By analyzing the effect of temperature on Z, rather than y, we are removing the influence of these factors.

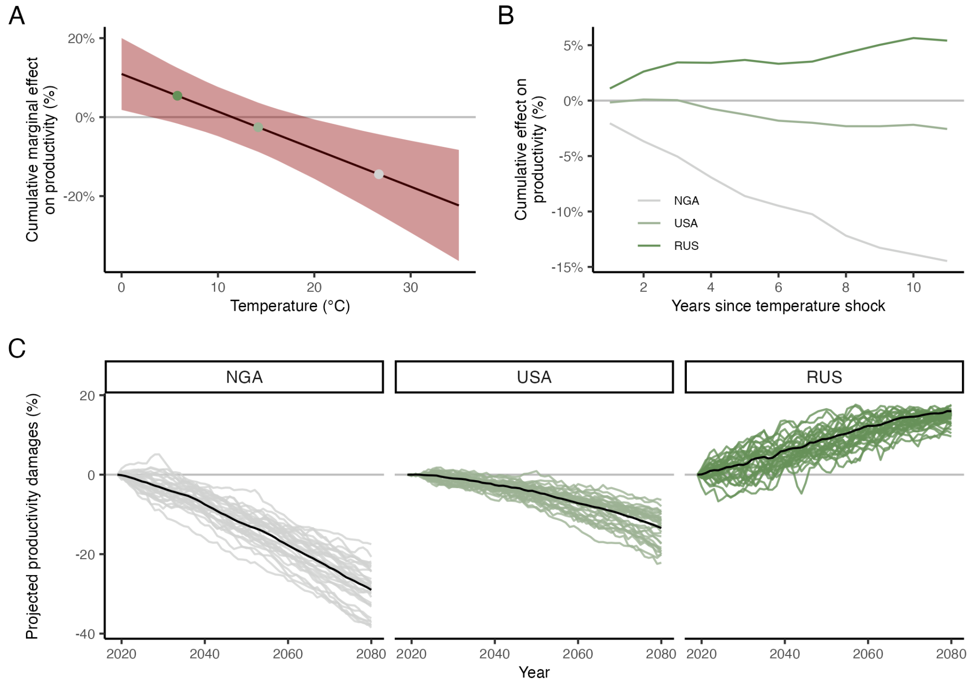


Figure 2: **Damage function.**
Panel A presents the estimated relationship between temperature and productivity from Equation (12), showing the cumulative marginal effect of a one-degree increase in temperature as a function of annual average temperature. Highlighted points indicate the estimated effects for a cold country (Russia), a temperate country (United States), and a hot country (Nigeria). Panel B reports the cumulative impulse response functions for these countries, illustrating the persistence and timing of productivity responses following a temperature shock. Panel C shows the projected paths of productivity for the same countries, given their projected temperatures across general circulation models (GCMs) in the CMIP6 ensemble. The black lines represent productivity changes associated with the mean projected warming across climate models in each period.

A shows the cumulative marginal effect of a one-degree increase in temperature as a function of baseline temperature. For relatively cold countries (such as Russia), where baseline temperatures lie below the estimated optimum of around 11.5C, moderate warming increases productivity. In contrast, countries with hotter baseline climates (such as Nigeria) experience substantial productivity losses as temperatures rise. Panel B decomposes the cumulative marginal effect shown in Panel A for countries with baseline temperatures corresponding to those of Nigeria, the United States, and Russia in 2019. The effects are highly persistent and grow rapidly for hot countries.

Constructing productivity shocks For each GCM (indexed by k), we accumulate the lagged marginal effects for a given path of temperature shocks, updating the climate system as follows,

$$C_{o,t}^{(k)} = C_{o,t-1}^{(k)} + \Delta C_{o,t}^{(k)}, \quad C_{o,0}^{(k)} = C_{o,2019} \, \forall k$$
$$d \ln Z_{o,t}^{(k)} = \sum_{\ell=0}^{H} \left(\hat{\beta}_{0,\ell} + 2\hat{\beta}_{1,\ell} C_{o,t-\ell}^{(k)} \right) \Delta C_{o,t-\ell}^{(k)}.$$

This procedure generates paths of productivity shocks for each country under each climate forecast.

Figure 2 shows the estimated productivity shocks implied by the CMIP temperature trajectories for Russia, USA, and Nigeria. Each line represents the evolution of productivity in a given country under a single CMIP6 general circulation model (GCM), based on the estimated temperature–productivity relationship in Equation (12). The dispersion across lines reflects uncertainty in future climate outcomes across GCMs, while the black line corresponds to the mean projected productivity path implied by the ensemble-average warming. Consistent with the damage function estimates, hot countries such as Nigeria experience persistent and increasing productivity losses over time, temperate countries such as the United States face moderate declines, and cold countries such as Russia exhibit gains under warming.

5 Results

5.1 The expected welfare impact of climate change

In Figure 3, we plot welfare loss by 2080 around the world (left panel). Consistent with a large literature (see, e.g., Cruz and Rossi-Hansberg, 2024), we find welfare gains in northern nations and substantial welfare losses in the global south. Appendix Figure A3 shows the aggregate welfare loss time series. Global welfare loss in 2080 is on average -18.7%, though this ranges from -33.3% to -8.3%. Using global PPP-adjusted GDP in 2019 as a dollar-denominated baseline (140.3 trillion USD), under the roughly 4.6 $GtCO_2$ emitted from 2019-2080 in RCP 8.5, and using a 1% discount rate (as our model abstracts from growth) and assuming damages end in 2080, this leads to a rough social cost of carbon (SCC) estimate of \$90 per tCO_2 – but the range of this SCC estimate across different CMIP projections goes from \$14 to \$192. In Appendix Figure A4, we plot the distribution of SCC estimates under different discount rates. The distribution of SCC estimates

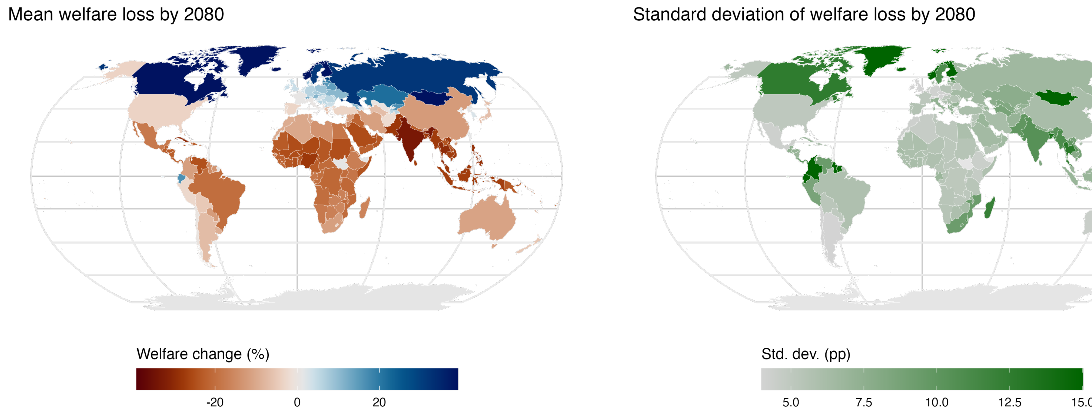


Figure 3: Left: welfare changes in 2080 in the mean RCP 8.5 forcing scenario. Right: the standard deviation of welfare outcomes under different CMIP projections in 2080.

is right skew and wider at lower discount rates, as welfare uncertainty is backloaded; we are more uncertain about damages in the future.

In the right panel, we show the standard deviation of welfare outcomes under different CMIP projections, colored by whether welfare changes are positive or negative at baseline. Welfare variance is higher for Northern Andean and Caribbean Rim nations. These nations are particularly vulnerable to ENSO events, a possible source of forecast uncertainty across CMIP models. We additionally find substantial welfare uncertainty for South and Central Asian countries. This uncertainty may stem from how forecast uncertainty on how climate change affects the Tibetan Plateau – and therefore the Asian Monsoon. Gains in high-latitude nations like Norway, Russia, and Mongolia, are also uncertain. Surprisingly, despite large predicted losses in sub-Saharan Africa, there is little uncertainty in projected welfare losses for most of the sub-continent.

In sum, welfare loss uncertainty is not perfectly explained by the size of the average gains or losses. To make this point clear, in the left panel of Figure 4 we show uncertainty in welfare loss against the mean welfare loss. The relationship is somewhat U-shaped, but absolute welfare change is not totally predictive of welfare loss variance. In the right panel, we plot welfare loss uncertainty against climate uncertainty, as given by the standard deviation of projected warming across CMIP-6 models by 2080. While welfare uncertainty is positively correlated with warming uncertainty, it is not perfectly predictive ($R^2 = 0.25$), which suggests that both the path of warming and nonlinear amplification through the damage function and general equilibrium play important roles in shaping the precision of our welfare loss estimates.

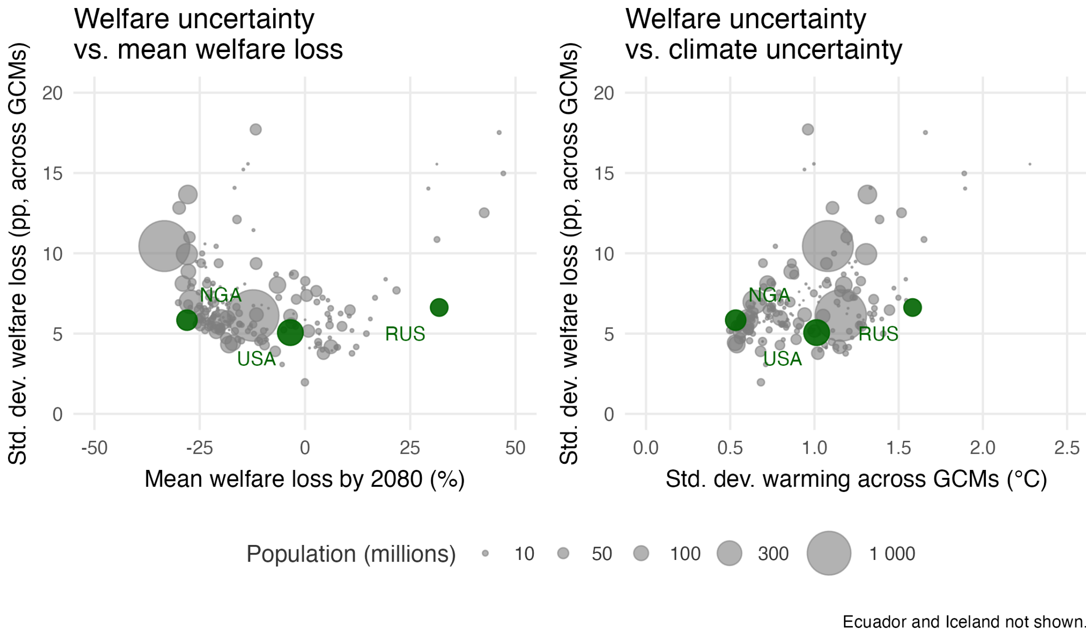


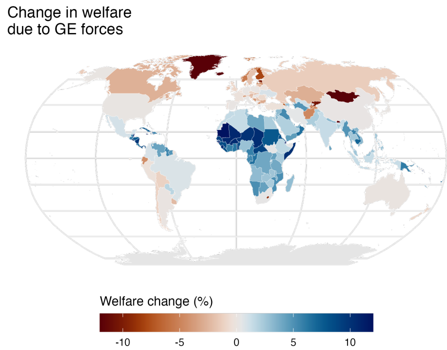
Figure 4: Left: Standard deviation of welfare loss by 2080 (in percentage points) against the mean welfare loss by 2080 across countries. Right: the same welfare uncertainty aginst the standard deviation of predicted warming in 2080 across CMIP-6 participating GCMs. Points are sized by their initial population. Ecuador and Iceland are omitted as they are outliers for welfare variance. Nigeria, USA, and Russia are highlighted.

We demonstrate the role trade and migration play in shaping expected welfare change in Figure 5. The left panel shows the change in welfare attributable to these general equilibrium forces over space.[13] There is substantial spatial heterogeneity: in sub-Saharan Africa and South and Southeast Asia, trade and migration attenuate the welfare loss from climate change implied by pure damages. Such countries suffer absolute productivity losses, but their terms-of-trade improve, as their relative productivity declines as well: their trading partners (Europe and China) face relatively small productivity changes from climate change. This gain also operates through emigration: as their migration option value rises, these nations depopulate, reducing their output, and subsequently putting upward pressure on the price of their exports on international markets.

The right panel of Figure 5 shows the role these forces play over time in three select countries. First, in Chad (TCD) and Greece (GRC), direct productivity losses are offset by these trade and migration gains, though in Greece, climate damages occur farther in the future than for Chad. This is because of the nonlinearity

[13]That is, it maps,

$$\mathbb{E}_k\left[\sum_{t=1}^{2080}\left(\boldsymbol{\mathcal{T}}_{t-1}+\boldsymbol{\mathcal{M}}_{t-1}\right)d\ln Z_t^{(k)}+\mathcal{L}_t^{(k)}\right].$$



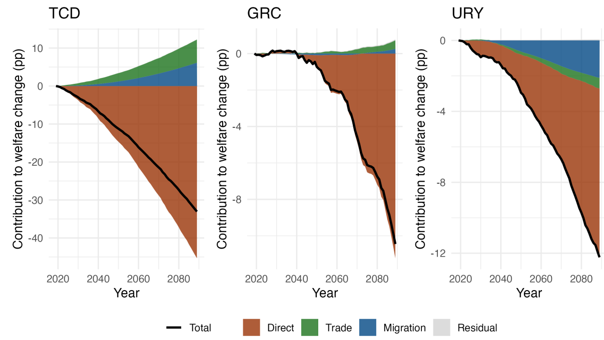


Figure 5: Left: the expected welfare impact of climate change in 2080 attributable to general equilibrium forces. Right: a time-series decomposition of these welfare changes for Chad (TCD), Greece (GRC) and Uruguay (URY).

of the damage function: local temperature must rise in Greece before the marginal effect of temperature shocks diminish productivity. By contrast, in Uruguay (URY), damages immediately begin to accrue, and general equilibrium forces amplify the welfare cost of climate change. Despite Uruguay suffering from absolute productivity losses rising local temperatures, its regional trading partners (e.g., Brazil) suffer larger climate damages. Consequently, Uruguay's relative productivity improves, diminishing its terms-of-trade. Moreover, Uruguay receives climate migrants from Brazil, further amplifying its welfare losses. Thus, our model demonstrates the extent to which the spatial correlation of climate shocks are filtered through the trade matrix is of first-order importance in projecting the welfare impact of climate change, a point made in Dingel and Meng (2025).

5.2 Climate risk over time and space

However, general equilibrium forces matter beyond the first order: they can amplify or attenuate the variance of climate shocks. In Figure 6, we plot the % change in the standard deviation of percent changes in welfare by 2080 measured across CMIP-6 models when accounting for trade and migration in addition to productivity shocks. In the left panel, we plot this across space: it is almost everywhere negative. General equilibrium forces dampen welfare variance relative to the variance of productivity shocks implied by the estimated damage function. Moreover, these general equilibrium forces play an important role in reducing welfare risk in sub-Saharan Africa and Central and Southeast Asia, where shock variance is substantial due to intermodel uncertainty in these regions. In the right panel, we plot the contribution to welfare variance (in squared

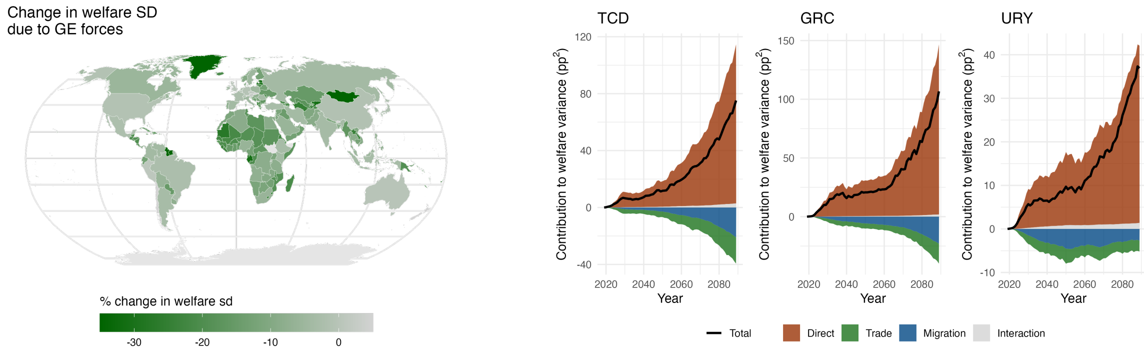


Figure 6: Left: The percent change in the variance of welfare changes by 2080 attributable to general equilibrium forces. Right: Decomposing the variance of welfare changes into direct (damage function), trade, and migration, and residual components.

percentage points) from damage function risk, and how this is filtered by the trade and migration matrices. In all three countries, general equilibrium forces play a sizable role in reducing climate risk over time.

In Figure 7, we whether the burden of climate risk is borne by the poorest nations today. While the standard deviation of projected warming by 2080 is largest for some of the richest countries today (left panel), this does not translate to lower welfare risk. Rather, we find a modest negative correlation between the level of development and climate risk: a 10% increase in 2019 GDP per capita is associated with a 0.1 fall in the standard deviation of projected warming by 2080.[14]

However, we do find that trade and adaptation matter most for some of the poorest regions in the world. In the left panel of Figure 8, we plot the percent change in the standard deviation of welfare changes by 2080 against baseline log GDP per capita. We find that the reduction in climate risk due to trade and migration is larger for poorer countries: a 1 log point decrease in log GDP per capita is associated with a 2 percentage point reduction in climate risk due to general equilibrium forces (Appendix Table A2). Absent adaptation, international inequality in climate risk, as measured by the Gini coefficient of the variance of log changes in welfare by 2080 is about 40% as large as international income inequality (as measured by the Gini coefficient for GDP per capita). Accounting for trade and migration reduces climate risk inequality by about 7 percentage points.

However, the level of development is not the only predictor of the capacity for trade and migration to lower climate risk. We additionally find that a country's trade openness at baseline matters in shaping the ability

[14] This correlation is population-weighted; T-statistic> 7, $R^2 = 0.22$.

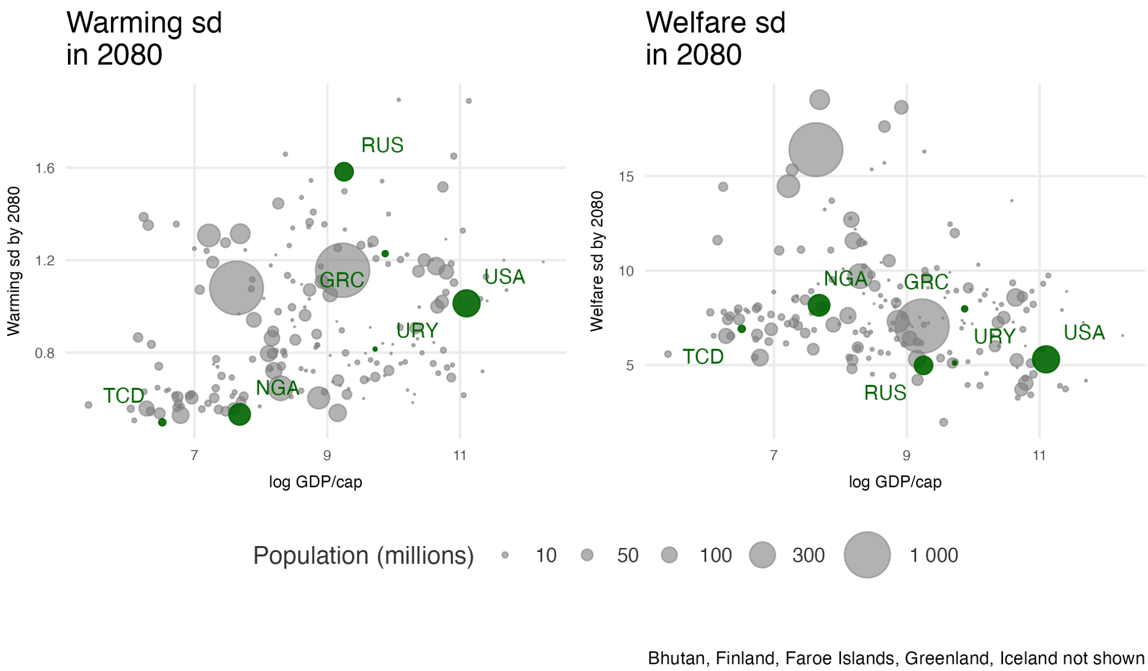


Figure 7: Left: standard deviation of degrees C of warming by 2080 across CMIP-6 models vs. log GDP/cap in 2019. Right: standard deviation of % changes in welfare by 2080 under different CMIP-6 model temperature forecasts vs. log GDP/cap. Small nations with large intermodel variation in projected warming like Iceland are omitted.

for economic geography forces to attenuate climate risk.[15] In the right panel of Figure 8, we show that it is in open economies that the reduction in climate risk due to trade and migration is largest. These countries have the largest capacity at baseline to diversify their exposure to climate risk. We find that a 1 log point change in trade openness is associated with a 5 percentage point reduction in climate risk due to trade and migration (see Appendix Table A2).

5.3 Jensen's correction

As the model is highly nonlinear, both due to the nonlinearity of the quadratic damage function with persistent effects and the structure of the model's equilibrium, it is plausible that expected welfare changes when integrating over future climate scenarios differ from the welfare changes computed using the mean temperature forecast across models due to Jensen's inequality.

[15] We measure trade openness as $\frac{1}{2}\frac{\text{Imports}+\text{Exports}}{\text{GDP}}$ in 2019. We take this from the World Development Indicators' NE.TRD.GNFS.ZS series.

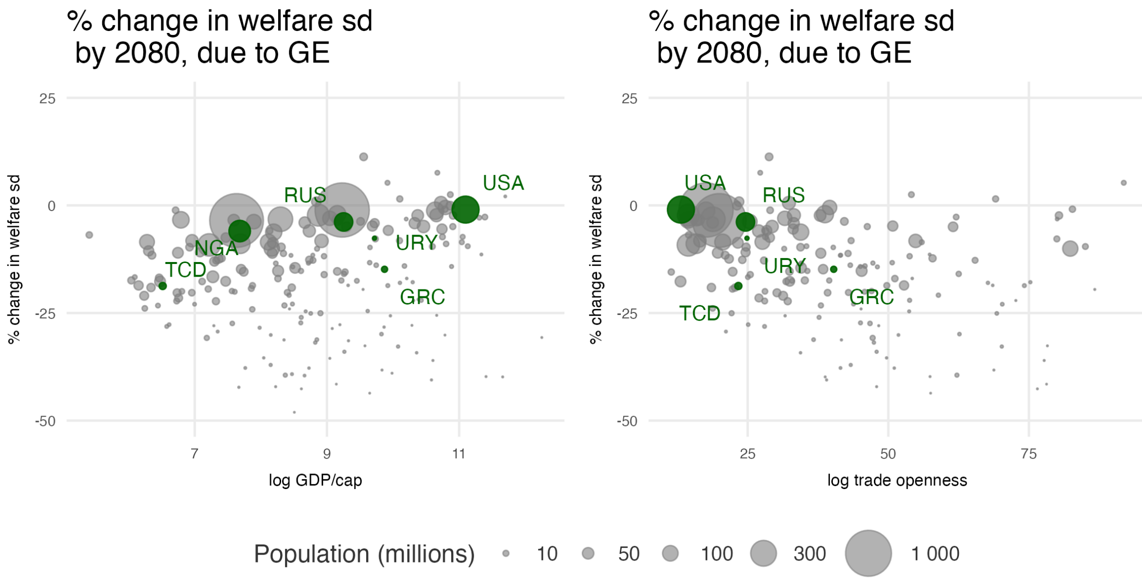


Figure 8: Left: the % change in the standard deviation of % changes in welfare by 2080 attributable to GE forces like trade and migration vs. log GDP/cap in 2019. Right: 2019 log GDP on the x-axis. In the left panel, Bhutan, Finland, Faroe Islands, Greenland, Iceland are not shown. In the right panel, Djibouti, Hong Kong, Ireland, Luxembourg, Malta, San Marino, and Singapore are not shown.

In the left panel of Figure 9, we plot the percentage point change adjustment to welfare changes by 2080 that comes from integrating over CMIP-6 models, instead of filtering the ensemble mean through the model, that is, we plot the Jensen's correction JC,

$$JC_{2080} = \mathbb{E}_k \left[\ln \mathcal{W}_{2080}^{(k)} \right] - \overline{\ln \mathcal{W}_{2080}}$$

where $\overline{\ln \mathcal{W}_{2080}}$ is the projected change in welfare evaluated at the mean CMIP-6 model temperature forecast. Using our linear decomposition, we are able to separate out the component of Jensen's correction attributable to the nonlinearity of the damage function from the nonlinearities from adaptation in the model, that is,

$$\begin{aligned} JC_{2080} = & \underbrace{\mathbb{E}_k \left[\sum_{t=0}^{2080} d \ln Z_t^{(k)} \right] - \sum_{t=0}^{2080} \overline{d \ln Z_t}}_{\text{damage function nonlinearity}} \\ & + \underbrace{\mathbb{E}_k \left[\sum_{t=0}^{2080} \left(\boldsymbol{\mathcal{T}}_{t-1}^{(k)} + \boldsymbol{\mathcal{M}}_{t-1}^{(k)} \right) d \ln Z_t^{(k)} \right] - \sum_{t=0}^{2080} \left(\overline{\boldsymbol{\mathcal{T}}}_{t-1} + \overline{\boldsymbol{\mathcal{M}}}_{t-1} \right) \overline{d \ln Z_t}}_{\text{general equilibrium adaptation nonlinearities}} . \end{aligned}$$

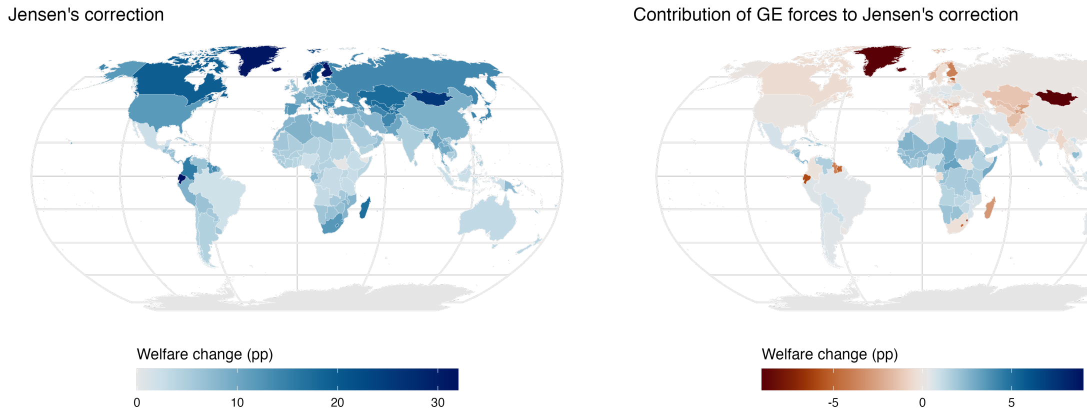


Figure 9: Left: percentage point welfare change adjustment when accounting for Jensen's inequality. Right: the percentage point change in Jensen's correction attributable to trade and migration.

The term $\mathbb{E}_k\left[d\ln Z_t^{(k)}\right] \neq \overline{d\ln Z_t}$ because $d\ln Z_t$ is a nonlinear function of the climate state $\{C_t^{(k)}\}_t$. In the right panel of Figure 9, we plot the component of Jensen's correction attributable to general equilibrium adaptation forces around the world. These are largest in sub-Saharan Africa, where adaptation forces play an important role in shaping the concavity of the welfare function and attenuating expected welfare losses when integrating over the distribution of future climate shocks.[16]

6 Conclusion

This paper develops a framework to understand how climate risk propagates through the global economy and how both a nonlinear, dynamic damage function, coupled with general-equilibrium forces—trade and migration—reshape both the expected cost and the variance of welfare under climate change. Motivated by the observation that climate projections remain deeply uncertain and that damages are highly nonlinear, we focus on how spatial linkages mediate not only the mean impact of warming but also the risk surrounding it. While prior research has shown that trade and migration can reallocate activity in response to climate change, we demonstrate that these same forces play a crucial role in redistributing and, in many cases, reducing welfare risk across space.

We estimate a nonlinear, dynamic damage function that maps changes in temperature to model-implied productivity. Using a long annual panel of country-level data, we exploit within-country temperature variation to

[16]In Appendix Figure A5 we plot this decomposition over time for six countries.

identify both the immediate and persistent impacts of warming on productivity. Our flexible distributed-lag specification allows for delayed responses, capturing persistent effects or local adaptation adjust productivity over the subsequent decade. This empirical foundation generates a rich and robust mapping from temperature to productivity that we embed directly into our quantified model, enabling stochastic simulations grounded in observed economic responses.

We then build and quantify a tractable dynamic spatial model that connects countries through trade and migration. These channels serve as margins of adaptation, allowing agents to reallocate resources in response to climate shocks, and as mechanisms of propagation, transmitting those shocks across borders and over time. The model incorporates risk in both the magnitude and geography of future warming by introducing stochastic fundamentals that evolve according to the estimated damage process. Within this environment, we derive analytical first- and second-order welfare approximations that decompose welfare changes into direct, trade, and migration components. The first-order terms describe how mean welfare responds to expected changes in productivity, while the second-order terms capture the curvature in welfare induced by nonlinear damages and general equilibrium feedback. Together, these decompositions provide a clear and tractable way to quantify how adaptation and exposure jointly shape the geography of climate risk.

Applying this framework globally, we decompose welfare outcomes into direct and general-equilibrium components for every country in the world. Consistent with past work, we find that welfare loss from climate change is most severe in poor, hot countries, while some rich, cold nations experience gains. Our first-order welfare decomposition shows that in much of sub-Saharan Africa and South and Southeast Asia, trade and migration attenuate direct welfare losses by facilitating reallocation toward less-affected regions. In other parts of the world, however, these forces can amplify damages through adverse terms-of-trade effects (when productivity falls relative to key trading partners) or migration flows that shift exposure across borders.

Examining climate risk, our results reveal a striking degree of spatial heterogeneity. Spatial climate risk inequality is almost half as large as global income inequality, with the burden falling on low income nations. Trade and migration substantially dampen welfare volatility almost everywhere, though to very different degrees. Highly integrated, low-income economies experience the greatest reductions in welfare variance due to trade and migration—up to 25–40 percent reductions in climate risk—compared to insular, higher-income nations like United States or Russia, who see minimal reductions in climate risk through these channels.

These patterns have direct policy implications. The benefits of global integration extend beyond traditional gains from trade and migration—they also provide a form of insurance against climate risk. More open economies are able to diversify exposure through international linkages, whereas closed economies remain more vulnerable to domestic climate variability. The geography of integration thus determines not only the expected cost of climate change but also the stability of welfare outcomes. Policies that facilitate smoother trade and migration flows, or that promote diversified regional production networks, can therefore mitigate both the mean and variance of global welfare losses.

Beyond the specific application to climate change, the framework developed in this paper can be used more broadly to study a wide range of shocks in quantitative spatial models. The theory provides a clean and general decomposition of welfare changes into direct, trade, and migration components, which together map productivity shocks into welfare outcomes through clearly interpretable mechanisms. This structure allows researchers to analyze how different types of shocks—such as technological innovation, policy interventions, geopolitical disruptions, or supply-chain disturbances—propagate across space and shape welfare both on average and its variance. In this sense, our model offers a flexible tool for studying how the geography of global integration mediates the welfare consequences of any stochastic disturbance that affects regional productivity.

Ultimately, this paper shows that exposure to climate risk is shaped as much by economic geography as by the climate system itself. Global linkages reduce the welfare risk for countries around the world, some more than others. The broader implication is that managing climate risk requires not only reducing emissions, but also building the institutional and economic linkages that allow regions to adapt collectively in an uncertain world.

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A Additional Tables and Figures

A.1 Data: descriptive plots

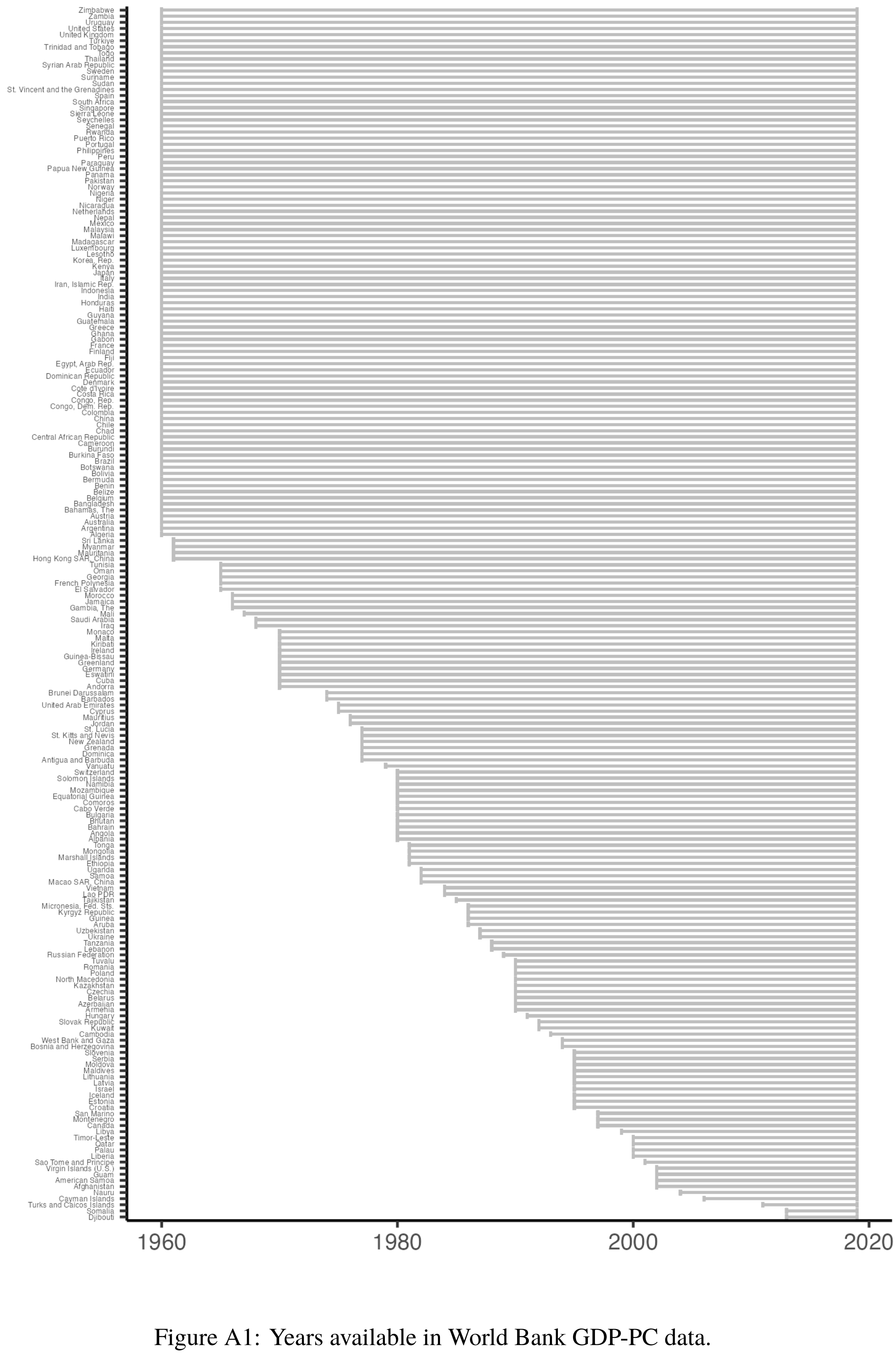

Figure A1: Years available in World Bank GDP-PC data.

A.2 Additional results

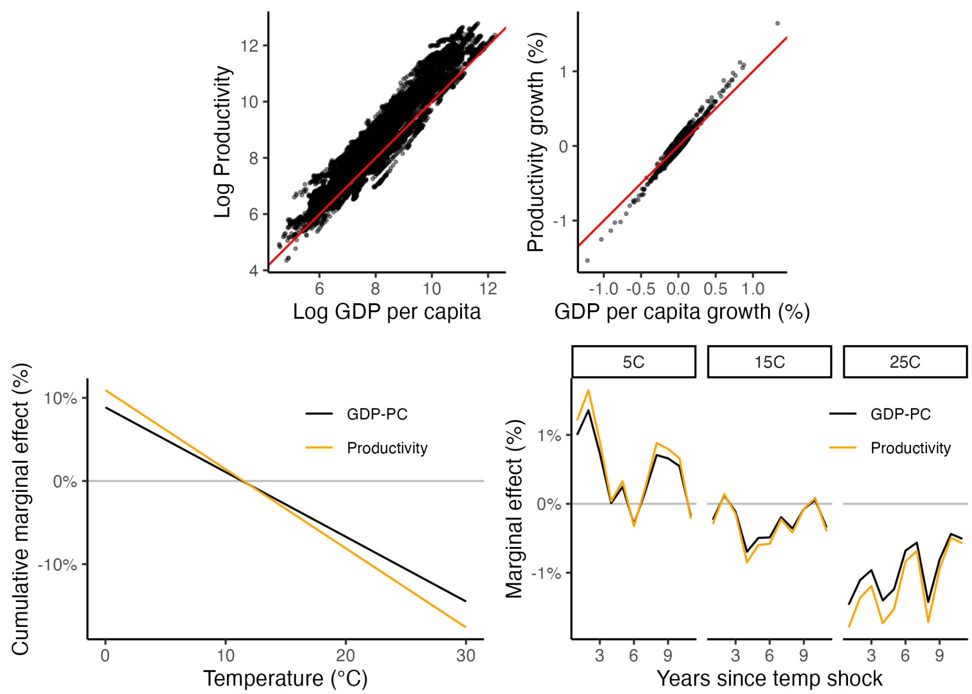

Figure A2: Comparison of damage function estimated on productivity to that estimated on GDP-PC.

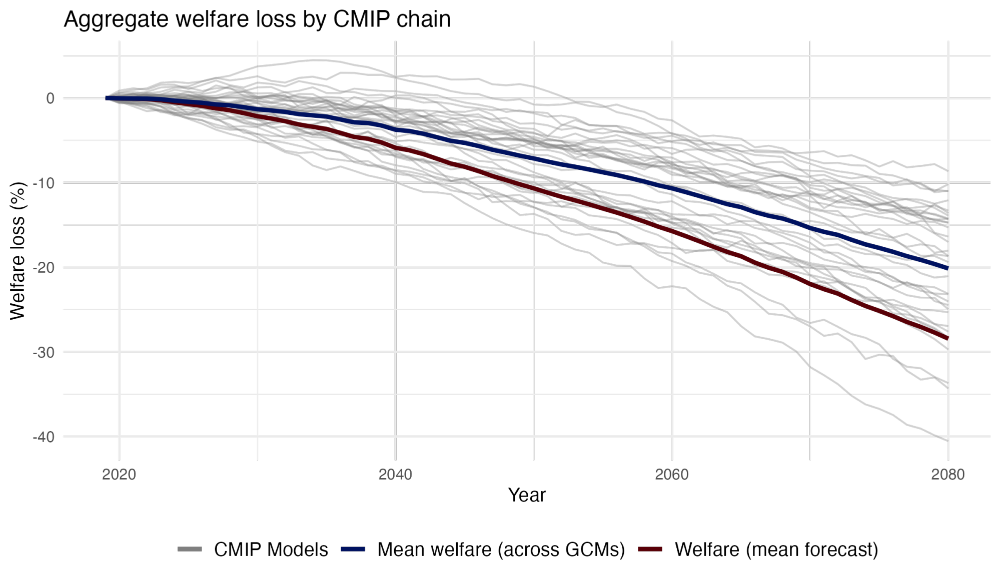

Figure A3: Global welfare loss $\ln \Upsilon_t - \ln \Upsilon_0$ under different CMIP models.

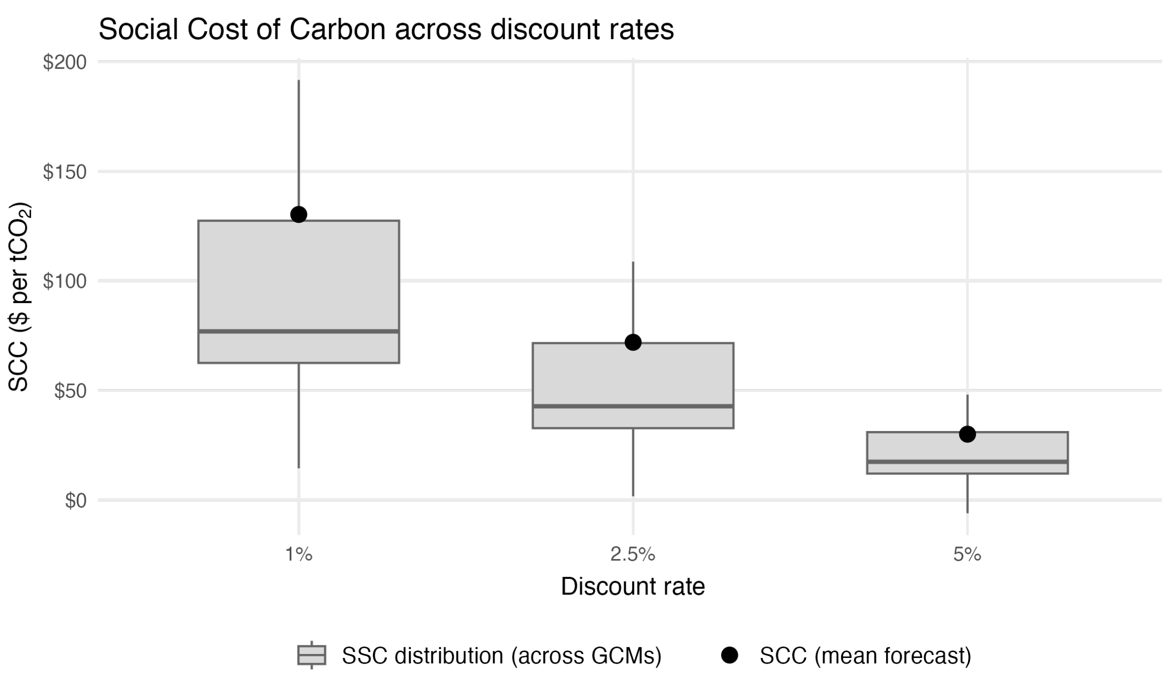

Figure A4: Distribution of SCC estimates for different discount rates.

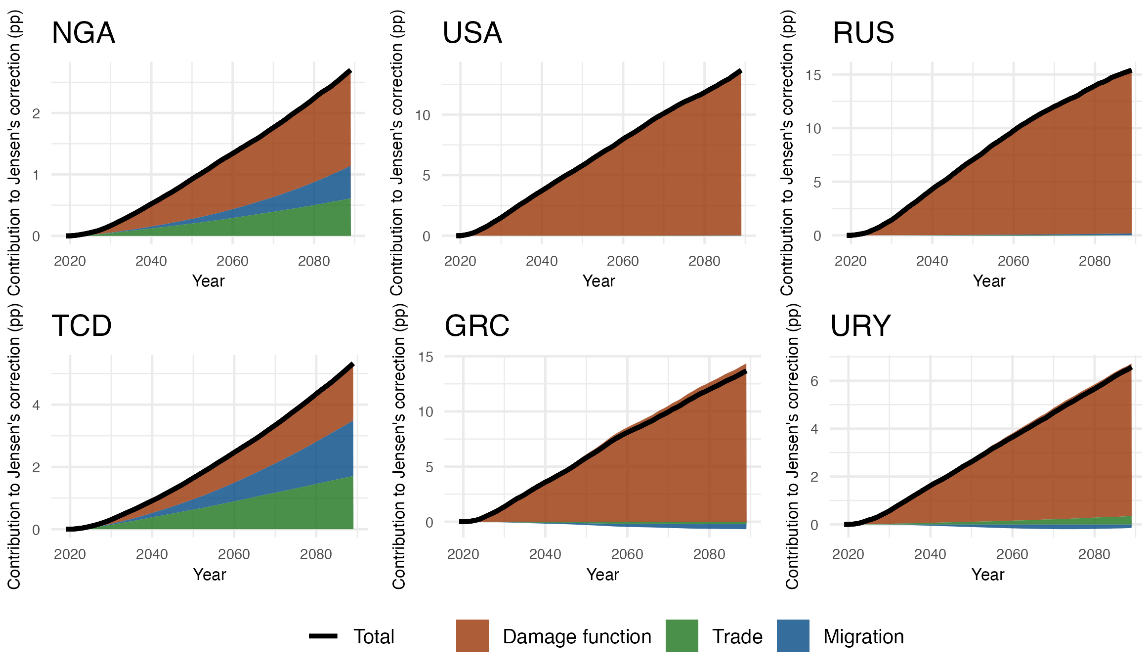

Figure A5: Caption

A.3 Additional Tables

	(1) Trade flows	(2) Migration flows
Log distance	-0.369	-1.009
	(0.015)	(0.065)
1(Colony of origin)	0.390	1.360
	(0.050)	(0.277)
1(Colony of destination)	0.310	1.231
	(0.057)	(0.266)
1(Common colonizer)	0.479	0.227
	(0.036)	(0.438)
1(Common legal origin)	0.045	-0.076
	(0.040)	(0.402)
1(Contiguous)	0.799	1.008
	(0.027)	(0.134)
1(Common language)	0.199	1.026
	(0.018)	(0.140)
1(Same country)	1.993	2.369
	(0.134)	(0.397)
1(Same region)	0.266	-0.064
	(0.036)	(0.135)
1(Both EU)	-0.105	-0.090
	(0.028)	(0.297)
Pseudo R-squared	0.981	0.998
N obs.	716,334	225,570

Table A1: Gravity regressions of trade and migration flows on bilateral cost shifters.

	(1)	(2)	(3)	(4)	(5)
Log trade openness	-5.402			-6.346	-5.347
	(1.742)			(1.483)	(1.091)
Log GDP/cap		1.044		2.357	2.267
		(0.575)		(0.555)	(0.545)
Temperature (2019, °C)			-1.480	-1.359	-0.002
			(0.220)	(0.212)	(0.180)
Std. Dev. Warming (2080)			-25.788	-25.748	0.034
			(4.110)	(3.950)	(3.111)
Observations	174	196	186	164	164
R^2	0.061	0.016	0.345	0.479	0.397
Population weights					✓

Table A2: Determinants of the percent reduction in climate risk attributable to general equilibrium forces (trade and migration). Robust standard errors in parentheses.

B Additional appendices

In each period t, the model is described by several equations,

- Technology and factor price determination,

$$Y_d = Z_d L_d, \quad w_d = p_d Z_d$$

- Preferences which generate,

$$V_d = A_d \frac{w_d}{P_d}, \quad P_d = \left(\sum_o (\tau_{od} p_o)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

  and a notion of welfare,

$$\mathcal{W}_o = \left(\sum_d (V_d/\mu_{od})^\epsilon \right)^{1/\epsilon} = V_o \left(\sum_d \mu_{od}^{-\epsilon} (V_d/V_o)^\epsilon \right)^{1/\epsilon}$$

  as well as migration choice probabilities,

$$M_{od} = \frac{(V_d/\mu_{od})^\epsilon}{\sum_{d'} (V_{d'}/\mu_{od'})^\epsilon}$$

- and period-t market clearing,

$$L_{d,t} = \sum_o M_{od} L_{o,t-1}$$

  and,

$$p_o Y_o = \sum_d \left(\frac{\tau_{od} p_o}{P_d} \right)^{1-\sigma} p_d Y_d$$

B.1 First-order decomposition

B.1.1 $d \ln Z$ shocks

Holding fixed amenity shocks $d \ln A_d = 0$, we start by log-differentiating welfare,

$$d \ln \mathcal{W}_o = \underbrace{d \ln V_o}_{\text{real income effect}} + \underbrace{\sum_d M_{od} \left(d \ln V_d - d \ln V_o \right)}_{\text{migration option value}}$$

Now,

$$d \ln V_d = \underbrace{d \ln Z_d}_{\text{direct effect}} + \underbrace{d \ln p_d - d \ln P_d}_{\text{trade effects}}$$

Now note,

$$d \ln P_d = \sum_o \underbrace{\left(\frac{\tau_{pd} p_o}{P_d} \right)^{1-\sigma}}_{\mathbf{T}_{od}} d \ln p_o$$

In matrix notation, this is $d \ln P = \mathbf{T}^\top d \ln p$. Taking stock, in matrix notation, we have,

$$d \ln V = d \ln Z + (\mathbf{I} - \mathbf{T}^\top) d \ln p$$

And so,

$$\begin{aligned} d \ln \mathcal{W} &= d \ln V + (\mathbf{M} - \mathbf{I}) d \ln V \\ &= \underbrace{d \ln Z}_{\text{direct effects}} + \underbrace{(\mathbf{I} - \mathbf{T}^\top) d \ln p}_{\text{trade effects}} + \underbrace{(\mathbf{M} - \mathbf{I}) \left[d \ln Z + (\mathbf{I} - \mathbf{T}^\top) d \ln p \right]}_{\text{migration effects}} \end{aligned}$$

What remains is to solve for $d \ln p$ as a function of $d \ln Z$. Doing so requires we understand the endogeneity of population in this model.

From the logit migration probabilities, we know,

$$d \ln M_{od} = \epsilon \left(d \ln V_d - \sum_{d'} M_{od'} d \ln V_{d'} \right)$$

So we can write,

$$dL_{d,t} = \sum_o M_{od,t}(d\ln M_{od})L_{o,t-1}$$

$$\begin{aligned}
dL_{d,t} &= \epsilon \sum_o M_{od,t}\left(d\ln V_d - \sum_{d'} M_{od'} d\ln V_{d'}\right) L_{o,t-1} \\
&= \epsilon \sum_o M_{od,t}(d\ln V_d)L_{o,t-1} - \epsilon \sum_o M_{od}\left(\sum_{d'} M_{od'} d\ln V_{d'}\right) L_{o,t-1} \\
&= \epsilon(d\ln V_d)L_{d,t} - \epsilon \sum_o M_{od}\left(\sum_{d'} M_{od'} d\ln V_{d'}\right) L_{o,t-1}
\end{aligned}$$

So then,

$$\implies d\ln L_{d,t} = \epsilon(d\ln V_d) - \epsilon \sum_o \frac{M_{od,t}L_{o,t-1}}{\sum_o M_{od,t}L_{o,t-1}} \sum_{d'} M_{od'} d\ln V_{d'}$$

Define $\mathbf{L}_{od,t} = \frac{M_{od,t}L_{o,t-1}}{\sum_{o'} M_{o'd,t}L_{o',t-1}}$: this is the share of persons in destination d who originated in o, so stacking, in vectors, we can write,

$$d\ln L = \epsilon(\mathbf{I} - \mathbf{L}^\top \mathbf{M})d\ln V$$

Now, let's analyze prices.

We start from,

$$p_oY_o = \sum_d \mathbf{T}_{od}p_dY_d$$

We know,

$$d\ln p_o + d\ln Y_o = \sum_d \mathbf{T}_{od}(d\ln p_d + d\ln Y_d)\frac{p_dY_d}{p_oY_o} + \sum_d (d\ln \mathbf{T}_{od})\mathbf{T}_{od}\frac{p_dY_d}{p_oY_o}$$

Defining $\mathbf{S}_{od} = \mathbf{T}_{od}\frac{p_dY_d}{p_oY_o}$ which is the share of GDP in o attributed to sales in d, we can write,

$$d\ln p_o + d\ln Y_o = \sum_d \mathbf{S}_{od}(d\ln p_d + d\ln Y_d) + \sum_d \mathbf{S}_{od}(d\ln \mathbf{T}_{od})$$

Now,

$$d\ln \mathbf{T}_{od} = (1-\sigma)\left(d\ln p_o - \sum_{o'} \mathbf{T}_{o'd} d\ln p_{o'}\right)$$

So stacking in matrix form,

$$d\ln p + d\ln Y = \mathbf{S}(d\ln p + d\ln Y) - (\sigma - 1)\left(\mathbf{I} - \mathbf{S}\mathbf{T}^\top\right) d\ln p$$

Rearranging,

$$\left(\sigma\mathbf{I} - \mathbf{S} - (\sigma-1)\mathbf{S}\mathbf{T}^\top\right) d\ln p = (\mathbf{S} - \mathbf{I})d\ln Y$$

Or,

$$d\ln p = \left(\sigma\mathbf{I} - \mathbf{S} - (\sigma-1)\mathbf{S}\mathbf{T}^\top\right)^{-1} (\mathbf{S} - \mathbf{I})d\ln Y$$

Now, note that,

$$\begin{aligned} d\ln Y &= d\ln Z + d\ln L \\ &= d\ln Z + \epsilon(\mathbf{I} - \mathbf{L}^\top \mathbf{M}) d\ln V \\ &= d\ln Z + \epsilon(\mathbf{I} - \mathbf{L}^\top \mathbf{M}) \left[d\ln Z + (\mathbf{I} - \mathbf{T}^\top) d\ln p \right] \\ &= \Big(\mathbf{I} + \epsilon(\mathbf{I} - \mathbf{L}^\top \mathbf{M}) \Big) d\ln Z + \epsilon(\mathbf{I} - \mathbf{L}^\top \mathbf{M})(\mathbf{I} - \mathbf{T}^\top) d\ln p \end{aligned}$$

This means that,

$$d\ln p = \Big(\sigma\mathbf{I} - \mathbf{S} - (\sigma - 1)\mathbf{S}\mathbf{T}^\top \Big)^{-1} (\mathbf{S} - \mathbf{I}) \left[\Big(\mathbf{I} + \epsilon(\mathbf{I} - \mathbf{L}^\top \mathbf{M}) \Big) d\ln Z + \epsilon(\mathbf{I} - \mathbf{L}^\top \mathbf{M})(\mathbf{I} - \mathbf{T}^\top) d\ln p \right]$$

Or,

$$\begin{aligned} &\left(\mathbf{I} - \Big(\sigma\mathbf{I} - \mathbf{S} - (\sigma - 1)\mathbf{S}\mathbf{T}^\top \Big)^{-1} (\mathbf{S} - \mathbf{I}) \epsilon (\mathbf{I} - \mathbf{L}^\top \mathbf{M})(\mathbf{I} - \mathbf{T}^\top) \right) d\ln p \\ &\quad = \Big(\sigma\mathbf{I} - \mathbf{S} - (\sigma - 1)\mathbf{S}\mathbf{T}^\top \Big)^{-1} (\mathbf{S} - \mathbf{I}) \Big(\mathbf{I} + \epsilon(\mathbf{I} - \mathbf{L}^\top \mathbf{M}) \Big) d\ln Z \end{aligned}$$

Which for simplicity, we can just call, $d\ln p = \mathbf{A}^Z d\ln Z$.

This means, then, that,

$$d\ln \mathcal{W} = \left(\underbrace{\mathbf{I}}_{\text{direct effects}} + \underbrace{(\mathbf{I} - \mathbf{T}^\top)\mathbf{A}^Z}_{\text{trade effects}} + \underbrace{(\mathbf{M} - \mathbf{I})}_{\text{migration effects}} + \underbrace{(\mathbf{M} - \mathbf{I})(\mathbf{I} - \mathbf{T}^\top)\mathbf{A}^Z}_{\text{trade-migration interaction}} \right) d\ln Z$$

This allows us to separate out variance in $\mathcal{W}$ driven by direct, trade, migration, and interaction effects.

B.1.2 $d\ln L^0$ shocks

Now, we want to analyze,

$$d\ln \mathcal{W} = (\mathbf{I} - \mathbf{T}^\top) d\ln p + (\mathbf{M} - \mathbf{I})(\mathbf{I} - \mathbf{T}^\top) d\ln p$$

when $d\ln Z = d\ln A = 0$ but $d\ln L^0$ is allowed to vary.

Now, as $L_d = \sum_o M_{od} L^0_o$, we have that,

$$dL_d = \sum_o M_{od}(d\ln M_{od}) L^0_o + \sum_o M_{od} dL^0_o$$

We know the first term is,

$$\implies d\ln L_{d,t} = \epsilon(d\ln V_d) - \epsilon \sum_o \frac{M_{od,t} L_{o,t-1}}{\sum_o M_{od,t} L_{o,t-1}} \sum_{d'} M_{od'} d\ln V_{d'}$$

the second term can be written as,

$$\sum_o \frac{M_{od}L^0_o}{\sum_{o'} M_{o'd}L^0_{o'}} d\ln L^0_o$$

Which gives,

$$d\ln L = \epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M})d\ln V + \mathbf{L}^\top d\ln L^0$$

where of course, $d\ln V = (\mathbf{I} - \mathbf{T}^\top)d\ln p$.

So, as $d\ln Z = 0$,

$$d\ln Y = \epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M})(\mathbf{I} - \mathbf{T}^\top)d\ln p + \mathbf{L}^\top d\ln L^0$$

And we know,

$$d\ln p = \left(\sigma\mathbf{I} - \mathbf{S} - (\sigma-1)\mathbf{S}\mathbf{T}^\top\right)^{-1}(\mathbf{S} - \mathbf{I})d\ln Y$$

So,

$$\begin{aligned} d\ln p &= \left(\sigma\mathbf{I} - \mathbf{S} - (\sigma-1)\mathbf{S}\mathbf{T}^\top\right)^{-1}(\mathbf{S} - \mathbf{I})d\ln Y \\ &= \left(\sigma\mathbf{I} - \mathbf{S} - (\sigma-1)\mathbf{S}\mathbf{T}^\top\right)^{-1}(\mathbf{S} - \mathbf{I})\left[\epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M})(\mathbf{I} - \mathbf{T}^\top)d\ln p + \mathbf{L}^\top d\ln L^0\right] \end{aligned}$$

So,

$$d\ln p = \left(\mathbf{I} - \left(\sigma\mathbf{I} - \mathbf{S} - (\sigma-1)\mathbf{S}\mathbf{T}^\top\right)^{-1}(\mathbf{S} - \mathbf{I})\right)^{-1}\left(\sigma\mathbf{I} - \mathbf{S} - (\sigma-1)\mathbf{S}\mathbf{T}^\top\right)^{-1}(\mathbf{S} - \mathbf{I})\mathbf{L}^\top d\ln L^0$$

B.2 Joint shocks and the accumulation of welfare effects

Recall that,

$$d\ln L = \epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M})d\ln V + \mathbf{L}^\top d\ln L^0$$

and,

$$d\ln V = d\ln Z + (\mathbf{I} - \mathbf{T}^\top)d\ln p$$

and,

$$d\ln p = \left(\sigma\mathbf{I} - \mathbf{S} - (\sigma-1)\mathbf{S}\mathbf{T}^\top\right)^{-1}(\mathbf{S} - \mathbf{I})d\ln Y$$

where, $d\ln Y = d\ln Z + d\ln L$. Plugging in,

$$\begin{aligned} d\ln p &= \left(\sigma\mathbf{I} - \mathbf{S} - (\sigma-1)\mathbf{S}\mathbf{T}^\top\right)^{-1}(\mathbf{S} - \mathbf{I})\left[d\ln Z + d\ln L\right] \\ &= \left(\sigma\mathbf{I} - \mathbf{S} - (\sigma-1)\mathbf{S}\mathbf{T}^\top\right)^{-1}(\mathbf{S} - \mathbf{I})\left[d\ln Z + \epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M})d\ln V + \mathbf{L}^\top d\ln L^0\right] \\ &= \left(\sigma\mathbf{I} - \mathbf{S} - (\sigma-1)\mathbf{S}\mathbf{T}^\top\right)^{-1}(\mathbf{S} - \mathbf{I})\left[d\ln Z + \epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M})\left[d\ln Z + (\mathbf{I} - \mathbf{T}^\top)d\ln p\right] + \mathbf{L}^\top d\ln L^0\right] \end{aligned}$$

Simplifying, $\mathbf{Q} = \left(\sigma\mathbf{I} - \mathbf{S} - (\sigma - 1)\mathbf{S}\mathbf{T}^\top\right)^{-1}(\mathbf{S} - \mathbf{I})$, we have,

$$d\ln p = \mathbf{Q}\left[(\mathbf{I} + \epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M}))d\ln Z + \mathbf{L}^\top d\ln L^0 + \epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M})(\mathbf{I} - \mathbf{T}^\top)d\ln p\right]$$

So,

$$d\ln p = (\mathbf{I} - \mathbf{Q}\epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M})(\mathbf{I} - \mathbf{T}^\top))^{-1}\mathbf{Q}\left[(\mathbf{I} + \epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M}))d\ln Z + \mathbf{L}^\top d\ln L^0\right]$$

Call,

$$\mathbf{A}^Z \equiv (\mathbf{I} - \mathbf{Q}\epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M})(\mathbf{I} - \mathbf{T}^\top))^{-1}\mathbf{Q}\left(\mathbf{I} + \epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M})\right)$$

and,

$$\mathbf{A}^L \equiv (\mathbf{I} - \mathbf{Q}\epsilon(\mathbf{I} - \mathbf{L}^\top\mathbf{M})(\mathbf{I} - \mathbf{T}^\top))^{-1}\mathbf{Q}\mathbf{L}^\top$$

So now,

$$d\ln W = \underbrace{\left[(\mathbf{I} + (\mathbf{I} - \mathbf{T}^\top)\mathbf{A}^Z + (\mathbf{M} - \mathbf{I})(\mathbf{I} + (\mathbf{I} - \mathbf{T}^\top)\mathbf{A}^Z)\right]}_{\equiv\mathbf{R}^Z} d\ln Z + \underbrace{\mathbf{M}(\mathbf{I} - \mathbf{T}^\top)\mathbf{A}^L}_{\equiv\mathbf{R}^L} d\ln L$$

So,

$$\ln\mathcal{W}_t - \mathcal{W}_{t-1} = \mathbf{R}^Z d\ln Z_t + \mathbf{R}^L d\ln L_t^0$$

$d\ln L$ is given as a function of the state, Z_{t-1}, as are the matrices $\mathbf{R}^Z, \mathbf{R}^L$ (mechanically, $Z_t = Z_{t-1}\exp(d\ln Z_t)$. and $L_t = \mathbf{M}_{t-1}L_{t-1}$).

So, given $\mathcal{W}_0$,

$$\ln\mathcal{W}_t - \ln\mathcal{W}_0 \approx \sum_{n=1}^{t} d\ln W_n$$

as $d\ln\mathcal{W}_n \approx \ln\mathcal{W}_n - \ln\mathcal{W}_{n-1}$.

So then,

$$\begin{aligned}
\ln\mathcal{W}_t - \ln\mathcal{W}_0 &\approx \sum_{n=1}^{t} d\ln W_n \\
&= \sum_{n=1}^{t} \mathbf{R}_{n-1}^Z d\ln Z_n + \mathbf{R}_{n-1}^L d\ln L_n^0 \\
&= \sum_{n=1}^{t} \mathbf{R}_{n-1}^Z d\ln Z_n + \underbrace{\sum_{n=1}^{t} \mathbf{R}_{n-1}^L d\ln L_n^0}_{\text{population convergence},\equiv\mathcal{L}_t} \\
&= \sum_{n=1}^{t} \left(\mathbf{I} + \boldsymbol{\mathcal{T}}_{n-1} + \boldsymbol{\mathcal{M}}_{n-1}\right) d\ln Z_n + \mathcal{L}_t
\end{aligned}$$

where $\boldsymbol{\mathcal{T}}_{n-1}$ capture trade propagation effects for time n shocks (and because of the linearization, that matrix is measurable at time $n-1$), and $\boldsymbol{\mathcal{M}}_{n-1}$ captures migration and migration-trade interaction effects.

B.3 Using the linearization for decomposition

For a path of shocks $d\ln Z_t^{(k)}$, we decompose the effects of each channel on welfare as, e.g.,

$$\begin{aligned}
\text{direct role}_t^{(k)} &= \frac{\sum_{n=1}^t d\ln Z_n^{(k)}}{\ln \mathcal{W}_t - \ln \mathcal{W}_0} \\
\text{trade role}_t^{(k)} &= \frac{\sum_{n=1}^t \boldsymbol{\mathcal{T}}_{n-1} d\ln Z_n^{(k)}}{\ln \mathcal{W}_t - \ln \mathcal{W}_0} \\
\text{migration role}_t^{(k)} &= \frac{\sum_{n=1}^t \boldsymbol{\mathcal{M}}_{n-1} d\ln Z_n^{(k)} + \mathcal{L}_t}{\ln \mathcal{W}_t - \ln \mathcal{W}_0}
\end{aligned}$$

We can also consider aggregate welfare, $\Upsilon_t = L_t^\top \mathcal{W}_t$. Decomposing any change, we have that,

$$\begin{aligned}
\Upsilon_t - \Upsilon_{t-1} &= \ln \sum_i L_{i,t}\mathcal{W}_{i,t} - \ln \sum_i L_{i,t-1}\mathcal{W}_{i,t-1} \\
\text{Törnqvist,} \quad &\approx \sum_i \frac{1}{2}(s_{it} + s_{i,t-1})(d\ln \mathcal{W}_{it} + d\ln L_{it}), \quad s_i = \frac{W_{it}L_{it}}{\Upsilon_{it}} \\
&\approx \sum_i \frac{1}{2}(s_{it} + s_{i,t-1})\left[(\mathbf{I} + \boldsymbol{\mathcal{T}}_{t-1} + \boldsymbol{\mathcal{M}}_{t-1})d\ln Z + \mathcal{L}_t\right]_i + \sum_i \frac{1}{2}(s_{it} + s_{i,t-1})(d\ln L_{it})
\end{aligned}$$

Now we also know $d\ln L_{it}$,

$$d\ln L_t = \epsilon(\mathbf{I} - \mathbf{L}^\top \mathbf{M})\left[\mathbf{I} + (\mathbf{I} - \mathbf{T}^\top)\mathbf{A}^Z\right] d\ln Z_t + \mathbf{L}^\top d\ln L_{t-1}$$

Similarly, we can decompose the variance across chains (k) through,

$$\text{trade variance share}_t = \frac{\mathrm{Var}_k\left(\sum_{n=1}^t \left(\mathbf{I} + \boldsymbol{\mathcal{T}}_{n-1}^{(k)}\right) d\ln Z_n^{(k)}\right) - \mathrm{Var}_k\left(\sum_{n=1}^t d\ln Z_n^{(k)}\right)}{\mathrm{Var}_k(\ln \mathcal{W}_t - \mathcal{W}_0)}$$

and similarly for migration share, and then the residual share captures are interactions and autocorrelations/covariances.

This works because, for, e.g., shock matrix Σ, we're basically attributing,

$$(I + T + M)\Sigma(I + T + M)' = \Sigma + \underbrace{T\Sigma + \Sigma T' + T\Sigma T'}_{\text{trade component}} + ...$$

And then we're accumulating the variances. All the autocorrelations from the accumulation are in the residual.

B.4 Second order effects

Start from,

$$d\ln\mathcal{W}_o = d\ln V_o + \sum_d M_{od}(d\ln V_d - d\ln V_o)$$

Differentiating again,

$$d^2\ln\mathcal{W}_o = d^2\ln V_o + \sum_d M_{od}(d\ln M_{od})(d\ln V_d - d\ln V_o) + \sum_{od} M_{od}(d^2\ln V_d/V_o)$$

And,

$$d^2\ln V_d = d^2\ln Z_o + d^2\ln p_d - \sum_o \mathbf{T}_{od}(d\ln\mathbf{T}_{od})(d\ln p_o) - \sum_o \mathbf{T}_{od}(d^2\ln p_o)$$

But $d^2\ln Z = 0$ by construction; it's the exogenous shock. Moreover, recall,

$$d\ln\mathbf{T}_{od} = -(\sigma-1)\left(d\ln p_o - \sum_{o'}\mathbf{T}_{o'd}d\ln p_{o'}\right)$$

So,

$$d^2\ln V_d = d^2\ln p_d + (\sigma-1)\sum_o \mathbf{T}_{od}\left(d\ln p_o - \sum_{o'}\mathbf{T}_{o'd}d\ln p_{o'}\right)(d\ln p_o) - \sum_o \mathbf{T}_{od}(d^2\ln p_o)$$

Writing,

$$\mathrm{Var}_{\mathbf{T}_{\cdot d}}(d\ln p_o) = \sum_o \mathbf{T}_{od}(d\ln p_o)^2 - \left(\sum_o \mathbf{T}_{od}d\ln p_o\right)^2$$

We have,

$$d^2\ln V_d = (\sigma-1)\mathrm{Var}_{\mathbf{T}_{\cdot d}}(d\ln p_o) - \sum_o \mathbf{T}_{od}(d^2\ln p_o - d^2\ln p_d)$$

i.e., "adaptation" through trade (price variance good, as we are maximizing) and second order price effects.

Now, to recall that,

$$d\ln M_{od} = \epsilon\left(d\ln V_d - \sum_{d'} M_{od'}d\ln V_{d'}\right)$$

So,

$$d^2\ln\mathcal{W}_o = d^2\ln V_o + \epsilon\sum_d M_{od}\left(d\ln V_d - \sum_{d'} M_{od'}d\ln V_{d'}\right)(d\ln V_d - d\ln V_o) + \sum_{od} M_{od}(d^2\ln V_d/V_o)$$

Similarly, defining,

$$\mathrm{Var}_{\mathbf{M}_{o\cdot}}(d\ln V_d/V_o) = \sum_d \mathbf{M}_{od}(d\ln(V_d/V_o))^2 - \left(\sum_d \mathbf{M}_{od}d\ln(V_d/V_o)\right)^2$$

We can write,

$$d^2 \ln \mathcal{W}_o = d^2 \ln V_o + \epsilon \text{Var}_{\mathbf{M}_{o\cdot}}(d \ln V_d/V_o) + \sum_{od} M_{od}(d^2 \ln V_d/V_o)$$

and then filling in for $d^2 \ln V_o$,

$$d^2 \ln \mathcal{W}_o = \underbrace{(\sigma-1)\text{Var}_{\mathbf{T}_{\cdot d}}(d \ln p_o)}_{\text{trade adaptation}} + \underbrace{\epsilon \text{Var}_{\mathbf{M}_{o\cdot}}(d \ln V_d/V_o)}_{\text{migration adaptation}} + \underbrace{\sum_{od} M_{od}(d^2 \ln V_d/V_o) - \sum_{o} \mathbf{T}_{od}(d^2 \ln p_o/p_d)}_{\text{second order price effects}}$$

And note to compute these, we have that, $d \ln p = \mathbf{A}\ d \ln Z$ and $d \ln V = \big(\mathbf{I} + (\mathbf{I} - \mathbf{T}^\top)\mathbf{A}\big)\, d \ln Z$.

C Inversion

C.1 Recovering productivity, Z_{it}

$$\begin{aligned}
w_i L_i &= \sum_j \left(\frac{\tau_{ij} w_i / Z_i}{P_j}\right)^{1-\sigma} w_j L_j \\
w_i^\sigma L_i Z_i^{1-\sigma} &= \sum_j (\tau_{ij})^{1-\sigma} \frac{w_j L_j}{P_j^{1-\sigma}} \\
Z_i^{1-\sigma} &= w_i^{-\sigma} L_i^{-1} \sum_j (\tau_{ij})^{1-\sigma} \frac{w_j L_j}{P_j^{1-\sigma}} \\
Z_i = w_i^{\frac{\sigma}{\sigma-1}} L_i^{\frac{1}{\sigma-1}} &\left(\sum_j (\tau_{ij})^{1-\sigma} \frac{w_j L_j}{P_j^{1-\sigma}}\right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

and,

$$P_j = \left(\sum_k \tau_{kj}^{1-\sigma} (w_k / Z_k)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

so the iteration is,

$$Z_i^{(n+1)} = w_i^{\frac{\sigma}{\sigma-1}} L_i^{\frac{1}{\sigma-1}} \left(\sum_j (\tau_{ij})^{1-\sigma} \frac{w_j L_j}{\left(\sum_k \tau_{kj}^{1-\sigma} \left(w_k / Z_k^{(n)}\right)^{1-\sigma}\right)}\right)^{\frac{1}{1-\sigma}}$$

As Z is only identified up to scale through this procedure, we impose the price in the USA to equal one, $p_{USA} = 1$ and we rescale $Z^{(n)}$ on each iteration so that $Z_{USA} = y_{USA}$ in every year.

Inverting Z requires that we have a complete panel of productivity. To form this panel and invert productivity, we interpolate GDP/cap for countries with missing values in the WDIs. To do so, we estimate,

$$\ln y_{it} = \beta \ln pop_i + \gamma_i \cdot y_{USA,t} + \alpha_i t + \xi_i + \xi_t + e_{it}$$

and use this regression to construct GDP/cap, y_{it} for missing observations. Appendix Figure A6 shows the fit of this interpolation exercise for countries with missing time series data.

Recovering 'stationary' amenities

The migration shares equation gives us,

$$L_j = \sum_i \mu_{ij}^{-\epsilon} \left(\frac{A_j v_j}{\sum_k (\mu_{ik} A_k v_k)} \right)^{\epsilon} L_i$$

where $v_j = w_j / P_j$, which can be computed knowing the inverted Z and σ. Rearranging,

$$A_j^{-\epsilon} = L_j^{-1} \sum_i \mu_{ij}^{-\epsilon} \left(\frac{A_j v_j}{\sum_k (\mu_{ik} A_k v_k)} \right)^{\epsilon} L_i$$

which suggests the updater,

$$A_j^{(n+1)} = L_j^{1/\epsilon} \left(\sum_i \frac{\mu_{ij}^{-\epsilon} v_j^{\epsilon}}{\sum_k \mu_{ik}^{-\epsilon} \left(A_k^{(n)} v_k \right)} L_i \right)^{-1/\epsilon}$$

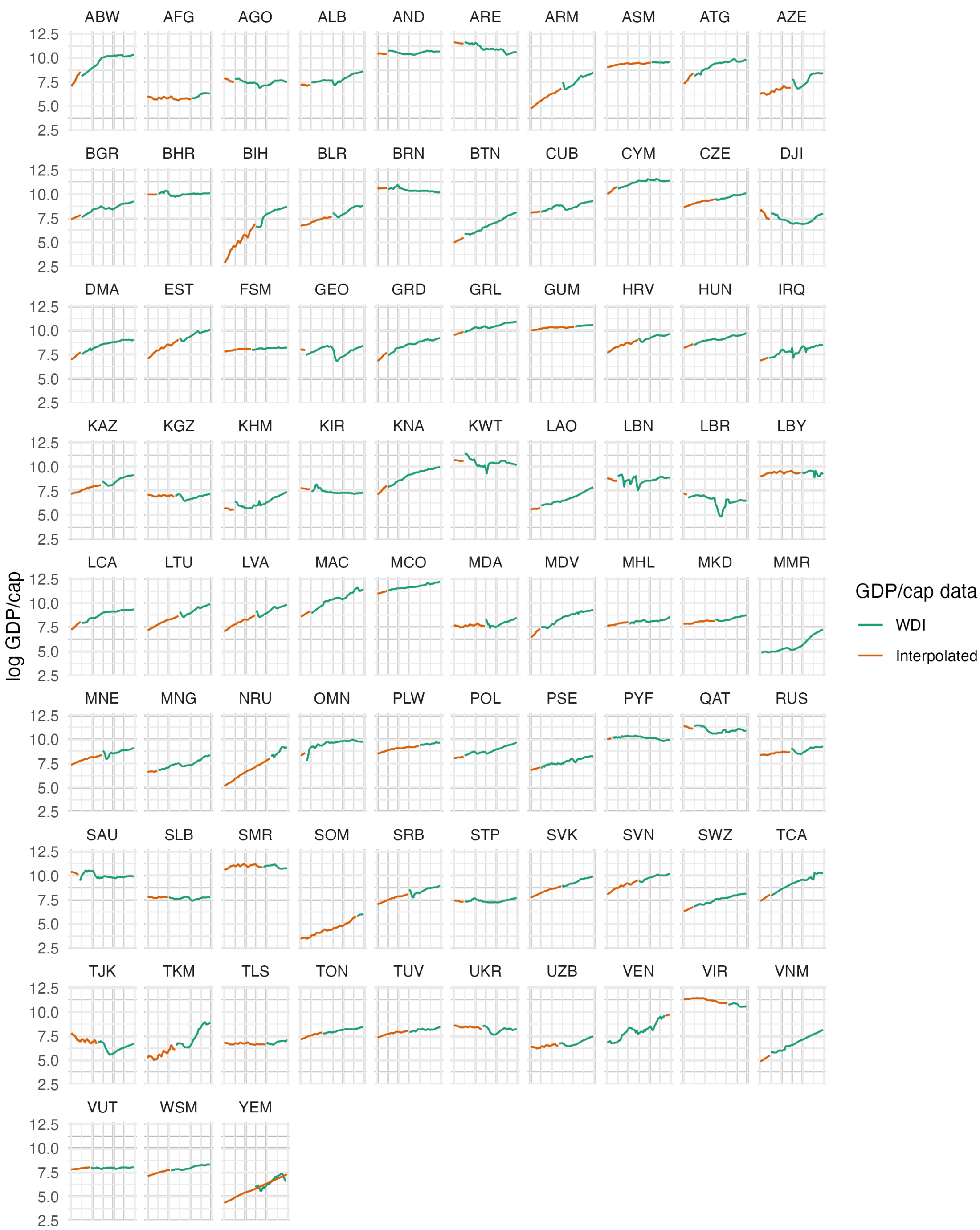

Figure A6: Interpolated and existing GDP/cap data. Time runs from 1960 to 2019.